TESIS – SS 142501

ESTIMASI PARAMETER PADA PEMODELAN *SPATIAL EXTREME VALUE* DENGAN PENDEKATAN COPULA

(Studi Kasus:Pemodelan Curah Hujan Ekstrem di Kabupaten Ngawi)

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SURABAYA
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PARAMETER ESTIMATION ON SPATIAL EXTREME VALUE MODEL WITH COPULA APPROACH

(Studi Kasus:Modelling Extreme Rainfall in Ngawi)

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Tesis disusun untuk memenuhi salah satu syarat memperoleh gelar Magister Sains (M.Si)
di
Institut Teknologi Sepuluh Nopember
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Tanggal Ujian : 10 Januari 2017
Periode Wisuda : Maret 2017

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Estimasi Parameter pada Pemodelan *Spatial Extreme Value* dengan Pendekatan Copula (Studi Kasus:Pemodelan Curah Hujan Ekstrem di Kabupaten Ngawi)

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ABSTRAK

Kejadian Ekstrem adalah suatu fenomena berskala pendek yang jarang terjadi dan biasanya jarang dapat dihindari, namun memberikan dampak yang cukup serius dari berbagai aspek kehidupan seperti curah hujan ekstrem. Studi mengenai pendugaan curah hujan ekstrem yang terjadi di suatu wilayah diperlukan untuk meminimalkan dampak buruk curah hujan ekstrem yang sering terjadi, sehingga petani dan *stakeholder* akan memiliki pengetahuan yang baik tentang curah hujan ekstrem. Untuk mendukung kebutuhan tersebut, diperlukan metode statistika yang dapat menjelaskan curah hujan ekstrem. *Extreme Value Theory* (EVT) merupakan salah satu metode statistika untuk mengidentifikasi kejadian ekstrem. Untuk data curah hujan, kelembaban, debit sungai, dan suhu termasuk data spasial yang merupakan data multivariat karena diamati berdasarkan lokasi. Oleh karena itu dikembangkan metode *Spatial Extreme Value*. Pada kasus multivariat, pendekatan yang sering digunakan adalah pendekatan copula dan proses *max-stable*. Penelitian tentang copula untuk kasus *Spatial Extreme Value* belum banyak dilakukan kajian. Oleh karena itu, penelitian ini membahas tentang Estimasi parameter pada pemodelan *Spatial Extreme Value* dengan pendekatan copula. Metode Estimasi parameter yang digunakan untuk *Spatial Extreme Value* dengan pendekatan copula adalah *Maximum Pairwise Likelihood Estimation* (MPLE). Penelitain ini diterapkan pada data curah hujan di salah satu sentra produksi padi Jawa Timur yaitu Kabupaten Ngawi. Data yang digunakan untuk menyusun model (verifikasi) dan estimasi parameter menggunakan data tahun 1990-2010, sedangkan untuk melakukan validasi model menggunakan data tahun 2010-2015.

Hasil penelitian menunjukkan bahwa estimasi parameter SEV dengan MPLE diperoleh penyelesaian yang tidak *close form*, sehingga dilanjutkan dengan metode iterasi Nelder-Mead**.** Nilai koefisien ekstermal berkisar antara 1,2-1,7 yang berarti bahwa data curah hujan ekstrem antar lokasi pos hujan di Kabupaten Ngawi terdapat dependensi. Kinerja prediksi curah hujan ekstrem dengan metode *Spatial Extreme Value* dengan pendekatan copula diperoleh RMSE sebesar 38,115.

Kata Kunci : Curah Hujan Ekstrim, Koefisien Ekstermal, *Spatial Extreme Value*, Copula.

Parameter Estimation on Spatial Extreme Value Model with Copula approach
(Case Study: Modelling Extreme Rainfall in Ngawi Regency)

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ABSTRACT

Extreme events is a short scale phenomenon rare, but quite a serious impact on the various aspects of life. Studies on the prediction of extreme rainfall that occurred in the region is needed to minimize the adverse effects of global climate change is often the case, so that farmers and stakeholders will have a good knowledge about the climate. Especially extreme rainfall events so early anticipation can be done, so that the production of rice plant can be maximized and losses can be minimized. To support these needs, the necessary statistical methods that can explain the extreme rainfall. Extreme Value Theory is a statistical method to identify extreme events. For the data of rainfall, snow, river flow, or temperature included as spatial data are multivariate data as observed in several locations, and therefore developed a method Spatial Extreme Value. In the case of multivariate data, the approach is often used Copula approach and process-maxstable. Parameter estimation method for use spatial extreme value with copula approach is maximum pairwise likelihood estimation. Therefore, this study discusses the Spatial modeling of Extreme Value with copula approach in one of East Java's rice production centers are Ngawi. The data used to construct the parameter estimation is the year 1990-2010, whereas the data for validation of model is the year 2010-2015. Value of extermal coefficient between 1,2-1,7 that indicated the data have dependence spatial. RMSE on spatial extreme value with copula approch with case study extreme rainfall in Ngawi is 38,155.

Keywords: Extreme Rainfall, Spatial Extreme Value, Extremal Coefficient, Copula

KATA PENGANTAR

Allah mengajarkan kita untuk bersyukur, satu kata yang jauh lebih luas maknanya daripada terimakasih. Alhamdulillah, hanya kata itu yang pantas penulis ucapkan atas kemurahan Allah *azza wa jalla* yang tiada henti sehingga penulis dapat menyelesaikan buku Tesis yang berjudul "**Estimasi Parameter pada Pemodelan *Spatial Extreme Value* dengan pendekatan Copula (Studi Kasus: Pemodelan Curah Hujan Ekstrem di Kabupaten Ngawi)**" dengan baik.

Penulis menyadari bahwa dalam penyusunan Tesis ini tidak terlepas dari bantuan dan dukungan dari berbagai pihak. Oleh karena itu, dengan segala kerendahan hati, penulis mengucapkan terima kasih banyak kepada:

1. Bapak Dr. Sutikno, M.Si dan Bapak Dr. Purhadi, M.Sc selaku dosen pembimbing yang dengan sabar memberikan bimbingan, ilmu, saran, dan waktu yang beliau luangkan untuk membimbing penulis.
2. Ibu Santi Wulan, M.Si, Ph.D dan Bapak Prof. Nur Iriawan, M.Ikom, Ph.D selaku dosen penguji atas kritik dan saran yang membangun dalam penyusunan Tesis ini.
3. Bapak Dr. Suhartono, M.Sc selaku Ketua Jurusan dan Bapak Dr. rer.pol. Heri Kuswanto, M.Si selaku Ketua Prodi S2 Statistika ITS yang telah memberikan banyak fasilitas untuk kelancaran penyelesaian Tesis ini.
4. Bapak Dr. Agus Suharsono, M.S selaku dosen wali, yang telah memberikan nasihat selama penulis menempuh studi.
5. Bapak dan Ibu dosen pengajar serta staf Jurusan Statistika FMIPA ITS Surabaya yang dengan tulus ikhlas telah memberikan bekal ilmu selama penulis melakukan studi.
6. Bapak, Ibu, Nenek, adik-adik tercinta dan semua keluarga penulis yang senantiasa mencurahkan segala rasa cinta dan semangatnya kepada penulis.
7. Rekan-rekan seperjuangan yang saling mendukung dan memberi semangat demi kelancaran menuju upacara wisuda bersama-sama, Statistika angkatan 2015 ganjil.

8. Teman-teman lintas angkatan Program Magister Jurusan Statistika, terimakasih atas kepeduliannya.
9. Semua pihak yang tidak dapat penulis sebutkan satu per satu.

Tak mampu penulis menyebut satu persatu semua pihak yang telah membantu dan mendukung. Tapi penulis juga memohon ketulusan maaf atas segala kesalahan diri penulis, juga pada pembaca semua. Sungguh itu murni datangnya dari penulis yang masih miskin ilmu dan pengalaman. Oleh karena itu, penulis mengharapkan kritik dan saran dari pembaca agar kedepannya lebih baik. Kini atas nikmat-Nya buku ini hadir, semoga bisa bermanfaat bagi kita semua. Amin.

Surabaya, Januari 2017

Penulis

DAFTAR ISI

DAFTAR TABEL

DAFTAR GAMBAR

DAFTAR LAMPIRAN

BAB 1
PENDAHULUAN

1.1 Latar Belakang

Perubahan iklim global dapat menyebabkan terjadinya kejadian ekstrem, seperti curah hujan ekstrem, suhu udara ekstrem, dan intensitas badai. Perubahan iklim global ini terjadi karena meningkatnya rata-rata temperatur dunia Frich, Alexander, Della-Marta, Gleason, Haylock, Tank dan Peterson (2002). Kejadian ekstrem adalah suatu fenomena berskala pendek yang jarang terjadi dan biasanya jarang dapat dihindari, namun memberikan dampak yang cukup serius dalam berbagai aspek kehidupan. Banyak permasalahan membutuhkan pengetahuan tentang perilaku nilai-nilai ekstrem, misalnya: kondisi infrastruktur, ketahanan pangan, penyedian air dan energi, kondisi tempat tinggal, dan transportasi dimana semua permasalahan tersebut sensitif terhadap tinggi atau rendahnya kondisi iklim dan cuaca. Sebagai contoh untuk kasus kondisi infrastruktur, curah hujan yang tinggi di suatu wilayah dapat mempengaruhi perancangan pembuatan sistem drainase, bendungan, waduk, dan jembatan. Perencanaan bendungan memerlukan data hidrologi yang meliputi data curah hujan. Data tersebut digunakan sebagai dasar perhitungan maupun perencanaan teknis dalam pembangunan bendungan agar dapat menampung air akibat curah hujan ekstrem (Surono dan Tunggul, 2015).

Jawa Timur merupakan salah satu provinsi yang diperhitungkan dalam memberikan kontribusi terhadap produksi padi secara nasional. Sekitar 17% produksi padi nasional berasal dari Jawa Timur (BPS,2016). Luas panen padi di Jawa Timur tahun 2009 mencapai 1.904.830 ha dengan produksi 11.259.085 ton. Namun, berdasarkan Dinas Pertanian Jawa Timur pada kwartalan pertama tahun 2010 lahan padi rusak akibat dampak kebanjiran mencapai nilai yang cukup signifikan yaitu sebesar 6.972,49 ha. Lima Kabupaten di Jawa Timur yang termasuk pemasok padi terbesar yaitu Kabupaten Jember, Bojonegoro, Lamongan, Banyuwangi dan Ngawi. Produktivitas padi di lima Kabupaten tersebut sebagai berikut, Lamongan mempunyai produksi sebesar 58,40 kw/Ha, Bojonegoro sebesar 56,28 kw/Ha, Jember sebesar 59,28 kw/Ha, Banyuwangi sebesar 62,81 kw/Ha dan Ngawi sebesar 63,60 kw/Ha (BPS, 2016). Dari kelima Kabupaten tersebut penghasil produksi padi terbesar adalah Kabupaten Ngawi. Selain pemasok padi terbesar, Kabupaten Ngawi juga merupakan daerah yang curah hujannya tinggi (Curah Hujan Ekstrem) saat musim hujan, sehingga rawan sekali terkena banjir (Hasan dan Utomo, 2009).

Curah Hujan ekstrem yaitu keadaan cuaca yang terjadi bila, jumlah hari hujan yang tercatat paling banyak melebihi harga rata-rata pada bulan yang bersangkutan di stasiun tersebut. Bila intensitas hujan terbesar dalam 1 (satu) jam selama periode 24 jam dan intensitas dalam 1 (satu) hari selama periode satu bulan yang melebihi rata-ratanya. Bila terjadi kecepatan angin > 45 km/jam dan suhu udara >35˚C atau <15˚C, serta curah hujan melebihi 100 mm/hari (BMKG, 2016). Curah hujan ekstrem menjadi perhatian khusus, karena peristiwa tersebut menimbulkan kerugian dalam sektor pertanian. Ketersediaan air hujan yang berlebihan mengakibatkan banjir dan terendamnya area pertanian, sehingga sawah menjadi rusak dan gagal panen. Studi mengenai pendugaan curah hujan ekstrem yang terjadi di suatu wilayah diperlukan untuk meminimalkan dampak buruk perubahan iklim global yang sering terjadi, sehingga petani dan *stakeholder* akan memiliki pengetahuan yang baik tentang iklim. Khususnya kejadian curah hujan ekstrem, agar antisipasi dini dapat dilakukan, sehingga produksi tanaman padi bisa dimaksimalkan dan kerugian bisa diminimalkan.

Beberapa penelitian dilakukan untuk memprediksi curah hujan ekstrem khususnya di Indonesia, antara lain Rosna (2014) , Anifah (2015) dan Hakim (2016) . Dalam penelitian tersebut, terdapat dua pendekatan yang digunakan untuk menentukan nilai ekstrem, yaitu *Block Maxima* (BM) dan *Peaks Over Threshold* (POT). Pendekatan BM menghasilkan distribusi nilai ekstrem berupa distribusi *Generalized Extreme Value* (GEV). Metode pendugaan parameter distribusi GEV diantaranya *Maksimum Likelihood* (ML) dan *Least Square* (LS). Penelitian tersebut juga membahas adanya kasus dependensi antar data ekstrem yang sering disebut data ekstrem stokastik.

Extreme Value Theory (EVT) merupakan salah satu metode statistika untuk mengidentifikasi kejadian ekstrem. EVT dikembangkan dari kasus univariat dengan kejadian ekstrem pada satu variabel dan sering diaplikasikan pada data saham. Untuk data-data curah hujan, salju, debit sungai, dan suhu termasuk sebagai data spasial yang merupakan data multivariat karena diamati pada beberapa lokasi, oleh karena itu dikembangkan metode *spatial extreme value*. Pada kasus data multivariat, pendekatan yang sering digunakan adalah pendekatan copula dan proses *max-stable* Cooley, Ciwewski, Edhart, Jeon, Mannshardt, Omolo dan Sun (2012).

Terdapat beberapa metode untuk menganalisis kejadian ekstrem dengan *spatial extreme value,* diantaranya adalah pendekatan copula yang dilakukan oleh Davison, Padoan, dan Ribatet (2012). Kemudian Cooley, Nychka, dan Naveau (2007) meneliti tentang presipitasi ekstrem spasial di Colorado dengan pendekatan *hierarchical Bayessian*. Selain itu, terdapat metode *Max Stable Process* (*MSP*) yang kembangkan oleh de Haan (1984) dan dikembangkan oleh beberapa peneliti lain seperti Schlather (2002), Kabluchko, Schlather, dan de Haan (2009). Aplikasi metode *max-stable* pada data curah hujan dapat ditemukan pada penelitian yang dilakukan oleh Buishand, de Haan, dan Zhou (2008), Smith dan Stephenson (2009), dan Davison dan Gholamrezaee (2010) pada data temperatur.

Copula merupakan salah satu metode statistika yang dapat menggambarkan hubungan antar variabel yang tidak terlalu ketat terhadap asumsi distribusi. Copula adalah suatu fungsi dari dua hubungan distribusi yang masing-masing mempunyai fungsi marginal distribusi (Nelsen,2005). Beberapa penelitian mengenai copula telah dilakukan, antara lain penelitian oleh Murteira dan Lourenco (2007) mengenai copula pada kasus kesehatan. Zhu, Ghosh, dan Goodwin (2008) menerapkan copula untuk memodelkan asuransi. Syahir (2011) menerapkan copula dibidang klimatologi. Ratih (2012) melihat dependensi dan memodelkan dengan Copula Regression untuk kasus pemodelan luas panen padi di Kabupaten Jember. Anisa (2015) melakukan pendekatan copula untuk analisis hubungan curah hujan dan indikator *El-Nino Southern Oscillation* di sentra produksi padi Jawa Timur. Selajutnya Sari (2013) mengidentifikasi dan menduga curah hujan ekstrem di 15 stasiun curah hujan di Kabupaten Indramayu. Hasil penelitian menunjukkan pendekatan dengan copula memberikan hasil yang tepat untuk data pengamatan ekstrem.

Estimasi parameter copula dapat dilakukan dengan berbagai cara, di antaranya: *maximum pairwise likelihood estimation* (MPLE), pendekatan Tau Kendall dan pendekatan Rho-Spearman. Kajian tentang estimasi parameter copula untuk kasus curah hujan ekstrem masih belum banyak dibahas. Oleh karena itu, pada penelitian ini dilakukan kajian mengenai estimasi parameter pada copula dengan *maximum pairwise likelihood estimation* (MPLE).

Penelitian ini melanjutkan empat penelitian sebelumnya dengan studi kasus pemodelan curah hujan ekstrem di Kabupaten Ngawi, Jawa Timur. Kabupaten Ngawi dipilih karena salah satu Kabupaten sentra produksi tanaman pangan (padi) di Jawa Timur dengan memberikan kontribusi sebesar 647.264 ton. Selain itu Ngawi merupakan daerah yang rawan banjir, sehingga apabila terjadi hujan ekstrem yang berkesinambungan tentu saja berpengaruh terhadap hasil panen atau produktivitas padi. Untuk itu dilakukan kajian di daerah Ngawi untuk mengetahui pola perilaku kejadian ekstrem. Oleh karena itu, penelitian ini membahas estimasi parameter pada pemodelan *Spatial Extreme Value* dengan pendekatan copula di salah satu sentra produksi padi Jawa Timur yaitu Kabupaten Ngawi.

1.2 Rumusan Masalah

Berdasarkan latar belakang, penelitian ini ingin mengetahui bagaimana kajian estimasi parameter pada pemodelan *Spatial Extreme Value* dengan pendekatan Copula. Disamping itu, bagaimana model curah hujan ekstrem di Kabupaten Ngawi berdasarkan pemodelan *Spatial Extreme Value* dengan pendekatan Copula.

1.3 Tujuan Penelitian

Tujuan yang ingin dicapai penelitian ini adalah sebagai berikut.

1. Mengkaji estimasi parameter pada pemodelan *Spatial Extreme Value* dengan pendekatan Copula.
2. Mendapat model curah hujan ekstrem di Kabupaten Ngawi menggunakan *Spatial Extreme Value* dengan pendekatan Copula.

1.4 Manfaat Penelitian

Manfaat yang diperoleh dari penelitian ini adalah menerapkan metode Statistika untuk menjelaskan kejadian ekstrem, sehingga dapat dijadikan pengetahuan dalam mengidentifikasi kejadian ekstrem di bidang Agroklimatologi. Selain itu diharapkan hasil penelitian dapat dimanfaatkan oleh Badan Meteorologi, Klimatologi dan Geofisika (BMKG) dalam pengembangan prediksi curah hujan ekstrem, sebagai antisipasi dini terjadinya bencana alam akibat curah hujan ekstrem.

1.5 Batasan Masalah

Penelitian ini menggunakan data curah hujan ekstrem harian di Kabupaten Ngawi Tahun 1990-2015. Copula yang digunakan dalam penelitian ini adalah copula gaussian.

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BAB 2
TINJAUAN PUSTAKA

2.1 *Extreme Value Theory*

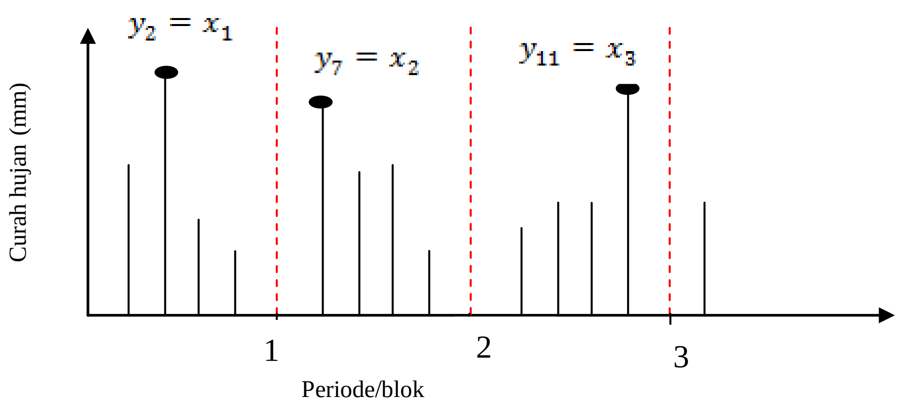
Kejadian Ekstrem merupakan hal yang penting untuk dikaji, pengkajian kejadian ekstrem digunakan untuk menentukan nilai probabilitas (maksimum atau minimum). *Extreme Value Theory* (EVT) merupakan salah satu metode statistika yang digunakan untuk mempelajari pola atau perilaku ekor (*tail*) dari distribusi tersebut, untuk dapat menentukan probabilitas nilai-nilai ekstremnya. Metode ini biasanya digunakan untuk menganalisis suatu kejadian yang bersifat ekstrem, dimana kejadian ini jarang terjadi dan berlangsung dalam waktu singkat namun memberikan dampak yang cukup besar. EVT digunakan untuk kasus univariat. Pengaplikasian EVT sudah dimulai lebih dari 50 tahun yang lalu (Coles, 2001) dalam berbagai bidang, seperti hidrologi, klimatologi, dan teori reliabilitas.

EVT dapat meramalkan terjadinya kejadian ekstrem pada data *heavytail* yang tidak dapat dilakukan dengan pendekatan standar (konvensional). Metode ini mampu menjelaskan kerugian kejadian ekstrem yang tidak dapat dimodelkan dengan pendekatan biasa. Sebagian besar data iklim memiliki ekor distribusi yang *heavytail*, yaitu ekor distribusi turun secara lambat bila dibandingkan dengan distribusi normal. Dampaknya adalah peluang terjadinya nilai ekstrem akan lebih besar daripada distribusi normal. Konsep dasar EVT adalah mengkaji perilaku stokastik variabel random baik maksimum maupun minimum (Kotz dan Nadarajah, 2000). Tujuan metode ini adalah untuk menentukan estimasi peluang kejadian ekstrem dengan memperhatikan ekor (*tail*) fungsi distribusi berdasarkan nilai-nilai ekstrem yang diperoleh.

Identifikasi nilai ekstrem dengan EVT dapat dilakukan dengan dua metode yaitu metode *Block Maxima* (BM) dan metode *Peaks Over Threshold* (POT). metode *Block Maxima* (BM) yaitu mengambil nilai maksimum dalam satu periode yang disebut sebagai blok dan metode *Peaks Over Threshold* (POT), yaitu mengambil nilai yang melewati suatu nilai *threshold* (McNeill, 1999). Pemilihan data ekstrem pada penelitian ini menggunakan metode BM.

2.2 Metode *Block Maxima*

Salah satu metode untuk mengidentifikasi nilai ekstrem adalah *Block Maxima* (BM). Metode BM adalah metode yang dapat mengidentifikasi nilai ekstrem berdasarkan nilai tertinggi data observasi yang dikelompokan berdasarkan periode tertentu yang disebut blok. Dalam metode ini, data pengamatan dibagi dalam blok-blok pada periode waktu tertentu, misalnya bulanan, triwulanan, semesteran, dan tahunan, kemudian setiap blok ditentukan nilai yang paling tinggi yang disebut sebagai nilai ekstrem untuk setiap blok. Nilai yang paling tinggi dimasukkan dalam sampel karena nilai inilah yang merupakan nilai ekstrem pada suatu periode tertentu. Gambar 2.1 menunjukkan ilustrasi pengambilan sampel dengan metode BM, dimana data curah hujan diamati mulai bulan (periode) pertama sampai keempat. Nilai observasi maksimum pada bulan pertama (blok pertama) adalah y_2, nilai y_2 dijadikan sampel ekstrem pada penelitian blok pertama dengan simbol dari sampel ekstrem blok pertama adalah x_1 sehingga $y_2 = x_1$. Untuk bulan kedua (blok kedua) nilai maksimum observasi adalah y_7, nilai y_7 dijadikan sampel ekstrem pada penelitian blok kedua dengan simbol dari sampel ekstrem blok kedua adalah x_2 sehingga $y_7 = x_2$. Untuk bulan ketiga (blok ketiga) nilai maksimum observasi adalah y_{11}, nilai y_{11} dijadikan sampel ekstrem pada penelitian blok ketiga dengan simbol dari sampel ekstrem blok ketiga adalah x_3 sehingga $y_{11} = x_3$ dan untuk bulan berikutnya pengambilan sampel dilakukan dengan cara yang sama.



Gambar 2.1 Ilustrasi *Block Maxima* (Gilli dan Kellezi, 2006)

Metode *block maxima* mengaplikasikan teorema Fisher dan Tippet (1928) dalam Gilli dan Kellezi (2006), dimana dalam teorema tersebut menyatakan bahwa data sampel nilai ekstrem yang diambil dengan metode BM akan mengikuti distribusi *Generalized Extreme Value* (GEV). Misalkan terdapat $X_1, X_2, ..., X_m$ merupakan variabel random dengan fungsi distribusi *F*, dan $Z_m = \max\{X_1, X_2, ..., X_m\}$ merupakan nilai maksimumnya. Jika Z_m konvergen ke salah satu limit *nondegenerate*, maka limit tersebut anggota keluarga parametrik oleh karena itu terdapat konstanta $\{a_m > 0\}$, $\{b_m\}$ dan *F* sehingga:

$$P\left\{\frac{Z_m - b_m}{a_m} \le x\right\} = F^n\left(a_m x + b_m\right) \to F\left(x\right)$$

Ketika $m \to \infty$, dengan *F* merupakan fungsi distribusi *nondegenerate*, maka *F* adalah salah satu keluarga dari distribusi Gumbel, Frechet dan *Reversed* Weibull (Gilli dan Kellezi, 2006). Menurut Mallor, Nualart, dan Omey (2009) *Generalized Extreme Value* (GEV) memiliki *cumulative distribution function* (CDF) seperti persamaan (2.1) sebagai berikut :

$$F(x;\mu,\sigma,\xi) = \begin{cases} \exp\left\{-\left[1+\xi\left(\dfrac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, -\infty < x < \infty, \xi \neq 0, -\infty < \mu < \infty, \sigma > 0 \\ \exp\left\{-\exp\left(-\dfrac{x-\mu}{\sigma}\right)\right\}, -\infty < x < \infty, \xi = 0, -\infty < \mu < \infty, \sigma > 0 \end{cases} \tag{2.1}$$

Probability distribution function (pdf) untuk distribusi GEV seperti persamaan (2.2).

$$f\left(x;\mu,\sigma,\xi\right) = \begin{cases} \dfrac{1}{\sigma}\left\{1+\xi\left(\dfrac{x-\mu}{\sigma}\right)\right\}^{-\frac{1}{\xi}-1} \exp\left\{-\left(1+\xi\left(\dfrac{x-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\} & ,\xi \neq 0,\ 1+\xi\left(\dfrac{x-\mu}{\sigma}\right) > 0 \\ \dfrac{1}{\sigma}\exp\left\{-\dfrac{x-\mu}{\sigma}\right\}\exp\left\{-\exp\left(-\dfrac{x-\mu}{\sigma}\right)\right\}, & \xi = 0 \end{cases} \tag{2.2}$$

dengan

x adalah nilai ekstrim yang diperoleh dari *block maxima* dengan $-\infty < x < \infty$

μ adalah parameter lokasi (*location*) dengan $-\infty<\mu<\infty$

σ adalah parameter skala (*scale*) dengan $\sigma >0$

ξ adalah parameter bentuk (*shape*) dengan $-\infty < \xi < \infty$

Tipe distribusi GEV ada 3 macam yaitu Tipe 1 berdistribusi Gumbel, Tipe 2 berdistribusi Frechet, dan Tipe 3 berdistribusi *Reversed* Weibull yang memiliki CDF seperti didefinisikan pada persamaan (2.3) sampai persamaan (2.5) sebagai berikut:

a. Distribusi Gumbel (distribusi *extreme value* tipe I) untuk $\xi = 0$

$$F(x;\mu,\sigma) = \exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}, -\infty < x < \infty \tag{2.3}$$

b. Distribusi Frechet (distribusi *extreme value* tipe II) untuk $\xi > 0$

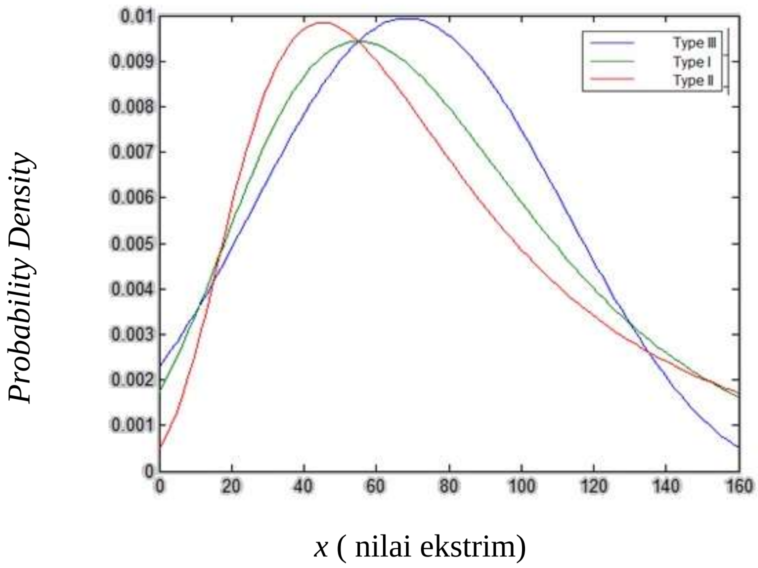
$$F(x;\mu,\sigma,\xi) = \begin{cases} 0 & , x \le \mu \\ \exp\left\{-\left(\dfrac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right\} & , x > \mu \end{cases} \tag{2.4}$$

c. Distribusi *Reversed* Weibull (distribusi *extreme value* tipe III) untuk $\xi < 0$

$$F(x;\mu,\sigma,\xi) = \begin{cases} \exp\left\{-\left(\dfrac{x-\mu}{\sigma}\right)^{\frac{1}{\xi}}\right\} & , x < \mu \\ 1 & , x \ge \mu \end{cases} \tag{2.5}$$

Dimana untuk semua tipe distribusi I, II, dan III $\sigma > 0$, dan $-\infty<\mu<\infty$. Bentuk distribusi GEV mengarah pada distribusi Gumbel untuk $\xi=0$, distribusi Frechet untuk $\xi>0$, dan distribusi *Reversed* Weibull untuk $\xi<0$. Nilai ξ merupakan parameter bentuk ekor (*tail*) dari ditribusi. Semakin besar nilai ξ, maka distribusi akan memiliki ekor yang semakin berat (*heavytail*) sehingga akan berdampak peluang terjadinya nilai ekstrem semakin besar. Menurut Finkenstadt dan Rootzen (2004) untuk parameter bentuk dengan $\xi = 0$ dikatakan "*medium tail*" ada juga menyebutnya "*exponensial tail*", untuk $\xi > 0$ dikatakan "*long tail*" dan untuk $\xi < 0$ dikatakan "*short tail*". Ketiga tipe distribusi GEV di atas menunjukkan bahwa distribusi yang memiliki ekor paling *heavytail* ialah distribusi Frechet ($\xi > 0$).

Ketiga distribusi ini memiliki bentuk ujung distribusi yang berbeda. Distribusi *Reversed* Weibull memiliki ujung distribusi yang terbatas, sedangkan distribusi Gumbel dan Frechet memiliki ujung distribusi yang tak terbatas. Selain itu, fungsi peluang *F* menurun secara eksponensial untuk distribusi Gumbel dan menurun secara polinomial untuk distribusi Frechet. Gambar 2.2 menunjukkan kurva ketiga distribusi nilai ekstrem.



Gambar 2.2 Bentuk pdf tipe distribusi GEV

Gambar 2.2 menunjukkan bentuk pdf dari 3 Tipe distribusi GEV yaitu distribusi Gumbel (type I), Frechet (type II), dan *Reversed* Weibull (type III) (Mallor, Nualart, Omey, 2009). Distribusi Gumbel kurva bersifat normal dan nilai μ tepat di 60, sedangkan untuk distribusi frechet kurva distribusinya miring ke kanan dan nilai μ berada di 40, sementara untuk distribusi *reversed* weibull kurva distribusinya miring ke kiri dan nilai μ berada di 80. Perbedaan kurva distribusi ini karena pengaruh nilai ξ, pada saat nilai $\xi > 0$ menyebabkan nilai modusnya bergeser ke arah kanan dan saat nilai $\xi < 0$ menyebabkan nilai modusnya bergeser ke arah kiri.

2.2.1 Estimasi Parameter Distribusi GEV Univariat dengan Maksimum Likelihood Estimation.

Estimasi perameter distribusi GEV dilakukan menggunakan metode *Maximum Likelihood Estimation* (MLE). Berikut adalah estimasi parameter untuk parameter $\hat{\mu}, \hat{\sigma},$ dan $\hat{\xi}$ dengan MLE menggunakan pdf distribusi GEV.

$$f(x;\mu,\sigma,\xi) = \begin{cases} \dfrac{1}{\sigma}\left\{1+\xi\left(\dfrac{x-\mu}{\sigma}\right)\right\}^{-\frac{1}{\xi}-1} \exp\left\{-\left[1+\xi\left(\dfrac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, \xi \neq 0 \\ \dfrac{1}{\sigma}\exp\left(-\dfrac{x-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\dfrac{x-\mu}{\sigma}\right)\right\} \qquad , \xi = 0 \end{cases} \tag{2.6}$$

Maka fungsi *likelihood* dengan $\xi \neq 0$ adalah:

$$L(\mu,\sigma,\xi \mid x_1 x_2 ... x_n) = \prod_{i=1}^{n} f(x_i;\mu,\sigma,\xi)$$
$$L(\mu,\sigma,\xi) = \prod_{i=1}^{n} \frac{1}{\sigma}\left\{1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right\}^{-\frac{1}{\xi}-1} \exp\left\{-\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\} \tag{2.7}$$

Selanjutnya memaksimumkan fungsi *likelihood*. Fungsi *ln likelihood* dibuat dengan cara membuat *ln* pada persamaan (2.7), seperti pada persamaan (2.8)

$$\ln\left(L(\mu,\sigma,\xi)\right) = \ln\left((\sigma)^{-n}\left[\sum_{i=1}^{n}\left\{1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right\}^{-\frac{1}{\xi}-1}\right]\exp\left\{-\sum_{i=1}^{n}\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}\right) \tag{2.8}$$

Selanjutnya Fungsi *ln likelihood* diturunkan terhadap $\hat{\mu},\hat{\sigma},$ dan $\hat{\xi}$ (2.8) kemudian disamadengankan 0, seperti persamaan (2.9) sampai (2.11).

$$\frac{\partial \ln\left(L(\mu,\sigma,\xi)\right)}{\partial\mu}=\left(\frac{1+\xi}{\sigma}\right)\sum_{i=1}^{n}\left\{1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right\}^{-1}-\frac{1}{\sigma}-\sum_{i=1}^{n}\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}-1}=0 \quad (2.9)$$

$$\frac{\partial \ln\left(L(\mu,\sigma,\xi)\right)}{\partial\sigma}=-\frac{n}{\sigma}+(1+\xi)\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma^2}\right)\left\{1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right\}^{-1}-\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma^2}\right)\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}-1}=0 \quad (2.10)$$

$$\frac{\partial \ln\left(L(\mu,\sigma,\xi)\right)}{\partial\xi}=-\frac{1}{\xi^2}\sum_{i=1}^{n}\ln\left\{1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right\}-\left(\frac{1}{\xi}+1\right)\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)^{-1}-$$

$$\sum_{i=1}^{n}\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}}\left[\frac{1}{\xi^2}\sum_{i=1}^{n}\ln\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)-\frac{1}{\xi}\sum_{i=1}^{n}\frac{\left(\frac{x_i-\mu}{\sigma}\right)}{\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)}\right]=0 \quad (2.11)$$

Dari hasil turunan pertama (2.9), (2.10) dan (2.11) diketahui bahwa turunan pertama fungsi *ln likelihood* terhadap masing-masing parameter tidak *closed form*, sehingga diperlukan pendekatan secara numerik untuk menyelesaikan persamaan tersebut. Analisis numerik yang digunakan *Nelder-Mead*. Karena meggunakan metode iterasi Nelder-Mead maka Fungsi *ln likelihood* dapat dituliskan $\ln\left(L(\mu,\sigma,\xi)\right)=\ell\{\boldsymbol{\psi}\}$ dimana $\boldsymbol{\psi}=(\mu,\sigma,\xi)$. Prosedur metode Nelder-Mead untuk memaksimumkan fungsi $\ell\{\boldsymbol{\psi}\}$ dimana $\ell\in R^3$ maka initial point yang digunakan yaitu ada sebanyak 3+1= 4 yaitu $\psi_1,\ldots,\psi_4$. Langkah-langkahnya sebagai berikut:

1. Substitusi nilai $\psi_1,\ldots,\psi_4$ ke dalam fungsi $\ell(\psi)$, kemudian diurutkan mulai nilai terbesar sampai terkecil $\ell(\psi_1)\geq\ell(\psi_2)\geq\cdots\geq\ell(\psi_4)$ sehingga ψ_1 disebut titik terbaik (*best*) dan ψ_4 disebut titik terburuk (*worst*).
2. Menentukan nilai ψ_o, yaitu nilai *centroid* (tengah) pada setiap *initial point* kecuali ψ_4.
3. Tahap *Reflection*

- Menentukan titik refleksi ψ_r dengan rumus $\psi_r = \psi_o + a(\psi_o - \psi_4)$, kemudian substitusi nilai ψ_r ke dalam fungsi $\ell(\psi)$ sehingga ada tiga kemungkinan kondisi yang dicapai oleh $\ell(\psi_r)$.
- Kondisi-1: jika ψ_r memenuhi kondisi $\ell(\psi_1) \geq \ell(\psi_r) > \ell(\psi_m)$, maka ψ_4= ψ_r dan kembali ke langkah-1.

4. Tahap *Expansion*
   - Kondisi-2: jika ψ_r memenuhi kondisi $\ell(\psi_r) > \ell(\psi_1)$, maka menentukan titik ekspansi ψ_e dengan rumus $\psi_e = \psi_o + b(\psi_o - \psi_4)$, kemudian substitusi nilai ψ_e ke dalam fungsi $\ell(\psi)$.
   - Selanjutnya jika titik ψ_e memenuhi kondisi $\ell(\psi_e) > \ell(\psi_1)$, maka ψ_4 = ψ_e dan kembali ke langkah-1. Sedangkan jika titik ψ_e tidak memenuhi kondisi tersebut, maka ψ_4= ψ_r, dan kembali ke langkah-1.
5. Tahap *Contraction*
   - Kondisi-3 : jika ψ_r memenuhi kondisi $\ell(\psi_r) \leq \ell(\psi_3)$, maka menentukan titik kontraksi ψ_c dengan rumus $\psi_c = \psi_o + c(\psi_o - \psi_4)$, kemudian substitusi nilai ψ_c ke dalam fungsi $\ell(\psi)$
   - Jika titik ψ_c memenuhi kondisi $\ell(\psi_c) > \ell(\psi_4)$, maka ψ_4= ψ_c dan kembali ke langkah-1.
6. Tahap *Reduction*

   Pada tahap ini jika ψ_r tidak memenuhi salah satu dari tiga kondisi tersebut, maka untuk setiap titik (kecuali titik terbaik ψ_1) diganti menggunakan rumus :

   $\psi_i = \psi_1 + d(\psi_i - \psi_1)$ dimana $i \in \{2,3,4\}$

Catatan: *a*, *b*, *c*, dan *d* adalah koefisien *reflection*, *expansion*, *contraction*, dan *shrink* dengan domain $a > 0$, $b > 1$, $0 < c < 1$, dan $0 < d < 1$. Nilai standar digunakan untuk koefisien-koefisien tersebut yaitu $a = 1$, $b = 2$, $c = -½$, dan $d = ½$ (Nelder dan Mead, 1965).

2.2.2 Uji Anderson Darling

Uji Anderson Darling adalah suatu uji yang digunakan untuk mengetaui apakah suatu data mengikuti distribusi tertentu (yang dihipotesiskan) atau tidak. Pengujian kecocokan distribusi GEV terhadap data ekstrem dapat dilakukan menggunakan uji Anderson Darling dengan prosedur (Engmann dan Cousineau, 2011) :

1. Uji Hipotesis :

   H_0 : $F(x) = F^*(x)$.(Data mengikuti distribusi teoritis $F^*(x)$)

   H_1: $F(x) \neq F^*(x)$.(Data tidak mengikuti distribusi teoritis $F^*(x)$)

2. Statistik Uji:

$$AD = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\Big(\ln\big(F^*(x_i)\big)\Big) + \ln\Big(1-\big(F^*(x_{n+1-i})\big)\Big) \quad (2.12)$$

   Keterangan:

   $F(x)$: fungsi distribusi kumulatif data sampel

   $F^*(x)$: fungsi distribusi kumulatif teoritis

   n : ukuran sampel

3. Penentuan kriteria uji

   Kriteria uji menolak H_0 jika nilai AD > nilai kritis yang ditentukan atau *p-value* $< \alpha$ (taraf signifikansi yang telah ditentukan). Nilai kritis ditentukan berdasarkan tabel Anderson Darling.

2.3 *Spatial Extreme Value*

Pada *Extreme Value Theory*, seringkali pemodelan univariat atau pada satu lokasi saja tidak cukup. Khususnya pada data *environment*, dimana kejadian ekstrem seperti hujan lebat, badai, salju, gempa bumi terjadi di beberapa lokasi berbeda yang berdekatan. Kejadian curah hujan ekstrem biasanya diukur berdasarkan lokasi sehingga pendekatan *extreme value theory* tidaklah cukup, oleh karena itu dibutuhkan pemodelan *spasial extreme value* untuk menduga curah hujan ekstrem. EVT dikembangkan dengan memasukkan unsur lokasi (*space)* atau yang dinamakan dengan *spatial extreme value*.

Salah satu pendekatan yang dapat digunakan untuk pemodelan *spasial extreme value* adalah melalui *multivariate extreme value*. Data spasial merupakan data multivariat karena diamati pada beberapa lokasi akibatnya ada asumsi tambahan yang harus dibuat, seperti asumsi dependensi spasial agar dapat bekerja pada model yang digunakan. Pada kasus ini, data ekstrem dari beberapa lokasi yang berbeda dipandang sebagai variabel multivariat atau berdistribusi multivariat. Misalkan $M(j,t)$ adalah data kejadian ekstrem pada lokasi ke j dan periode waktu ke t, pada domain spasial $D \subset R^2$. Distribusi dari $M(j,t)$ adalah:

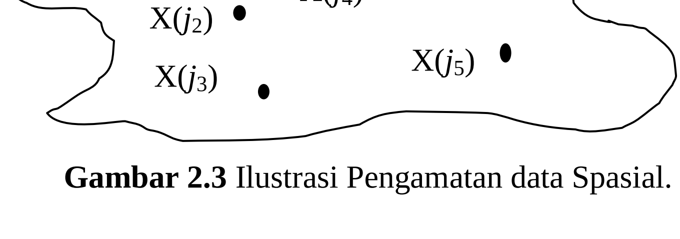
$$M(j,t) \sim GEV(\mu(j,t), \sigma(j,t), \xi(j,t)) \qquad (2.13)$$

dimana $\mu(j,t)$, $\sigma(j,t)$, dan $\xi(j,t)$ merupakan parameter lokasi, skala, dan bentuk distribusi GEV dimana t=1,2,...,T dan j=1,2,...,J. Dengan asumsi bahwa tiap komponen pada tiap lokasi berdistribusi GEV, selanjutnya dilakukan transformasi seperti pada persamaan (2.20). Dalam konsep spasial kejadian pada suatu lokasi yang berdekatan cenderung memiliki kemiripan atau memiliki hubungan yang cukup erat daripada kejadian pada lokasi yang lebih jauh.

Jika obyek yang diamati berupa titik, sangat banyak observasi yang mungkin berada dalam wilayah D. Obyek yang diukur pada region D dianggap bagian dari kumpulan obyek yang besar. Misalkan terdapat satu karakteristik atau variabel yang diukur pada titik yang berbeda dalam suatu lokasi dan waktu pengamatan diabaikan. Ada n observasi yang disimbolkan sebagai berikut

$$X(j_i) \text{ dimana } i=1,2,...,n \text{ dengan } j \in D$$

Gambar 2.3 menunjukkan ilustrasi data spasial pada suatu lokasi, ilustrasi data spasial yang diamati pada 5 titik lokasi. Pengamatan pada titik yang berdekatan, misalkan pada $X(j_1)$ dan $X(j_3)$ atau $X(j_2)$ dan $X(j_3)$ memiliki dependensi yang lebih besar dibandingkan pengamatan pada titik yang berjauhan.



Gambar 2.3 Ilustrasi Pengamatan data Spasial.

Salah satu pendekatan yang dapat digunakan untuk pemodelan *spatial extreme value* adalah melalui *multivariate extreme value*. Pada data multivariat, pendekatan yang sering digunakan adalah pendekatan copula dan proses *max-stable*.

2.4 Madogram

Konsep dasar madogram berasal dari semivariogram yang merupakan grafik antara semivariansi terhadap fungsi jarak. Semivariogram dapat digunakan untuk mengukur dependensi spasial. Hubungan kebergantungan spasial antara titik-titik lokasi turut ditentukan oleh jarak antar lokasi, semakin dekat suatu lokasi akan memiliki semivarian yang kecil dan berlaku sebaliknya. Konsep jarak yang digunakan yaitu konsep jarak *Euclid.* Semivariogram dapat didefinisikan pada persamaan berikut:

$$\gamma(h) = \frac{1}{2} E\left[U(j+h) - U(j)\right]^2 = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left[U(j_i + h) - U(j_i)\right]^2 \qquad (2.14)$$

dengan:

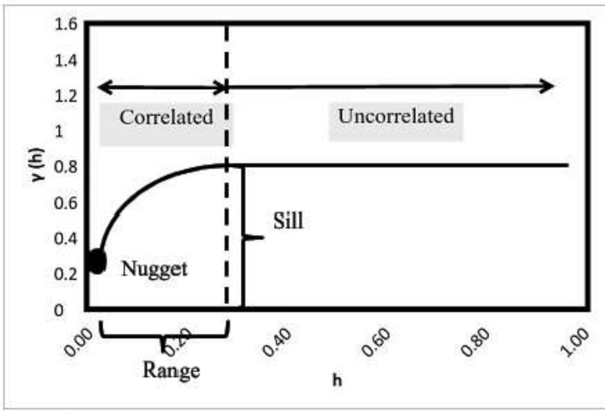
$\gamma(h)$: nilai semivariogam dengan jarak h

$U(j_i)$: nilai pengamatan di titik j_i

$U(j_i + h)$: nilai pengamatan di titik $(j_i + h)$

$N(h)$: banyaknya pasangan titik yang berjarak h.

Sebelum menentukan model semivariogram, perlu dilakukan pendugaan terhadap parameter-parameter semivariogram. Parameter tersebut diduga berdasarkan plot semivariogram yang dihasilkan. Plot semivariogram ditunjukkan pada Gambar 2.4. parameter yang diperlukan untuk mendeskripsikan plot semivariogram yaitu:



Gambar 2.4 Ilustrasi Plot Semivariogram

1. *Nugget Effect* (C0)

   Nugget Effect adalah pendekatan nilai semivariogram pada jarak di sekitar nol.

2. *Sill* (C_0 + C)

   Sill merupakan sebuah nilai tertentu yang konstan yang dimiliki oleh semivariogram untuk jarak tertentu sampai dengan jarak yang tidak terhingga atau nilai semivariogram dimana menunjukkan sudah tidak terdapat lagi korelasi antar data. Apabila *h* semakin besar maka korelasi pada dua titik dengan jarak *h* dapat diabaikan. Dalam kasus seperti ini nilai semivariogram $\gamma(h) = \sigma^2$ sehingga partial sill (C) dalam plot semivariogram adalah varians.

3. *Range* (a)

   Range merupakan jarak maksimum dimana masih terdapat korelasi an tar data.

Semivariogram hanya bisa digunakan pada distribusi data yang memiliki ekor pendek yang artinya tidak bisa digunakan dalam kasus data *extreme*. Untuk mengatasi hal itu, Cooley (2006) menggunakan semivariogram orde pertama yang disebut madogram yang bisa digunakan untuk data ekstrem. Teori tentang madogram telah dipelajari oleh Matheron pada tahun 1987 dalam Cooley (2006) yang didefinisikan sebagai berikut:

$$v(h)=\frac{1}{2}E\left[U(j+h)-U(j)\right] \tag{2.15}$$

Madogram mengharuskan momen pertama terhingga yang tidak selalu terjadi pada kasus ekstrem, untuk mengatasinya Cooley (2006) memperkenalkan madogram yang mentransformasi *variabel random* menggunakan distribusi GEV. Fungsi *F* berdistribusi GEV, sehingga *F*- madogramnya sebagai berikut:

$$v(h)=\frac{1}{2}E\left[F\left(U(j+h)\right)-F\left(U(j)\right)\right] \tag{2.16}$$

Dalam proses penentuan pola semivariogram, terkadang melibatkan banyak titik pada plot semivariogram sehingga sulit untuk melihat pola tertentu. Untuk mengatasi hal tersebut, maka madogram dikelompokkan berdasarkan kesamaan jarak. Sehingga, perhitungan *F*- madogram dapat dinyatakan sebagai berikut:

$$\hat{v}_F(h)=\frac{1}{2\left|N(h)\right|}\sum_{i=1}^{N(h)}\left[F\left(U(j_i+h)\right)-F\left(U(j_i)\right)\right] \tag{2.17}$$

dengan $\hat{v}_F(h)$ adalah *F*-madogram pada lag *h*, j_i adalah lokasi titik pengamatan, $U(j_i)$ adalah nilai pengamatan pada lokasi ke j_i, *h* adalah jarak antara dua lokasi, (j_i, j_i+h) adalah pasangan data yang berjarak *h*, dan *N(h)* adalah banyaknya pasangan lokasi yang berjarak *h*. Koefisien ekstermal dan *F*-*m*adogram mempunyai hubungan yang sangat kuat ditunjukkan sebagai berikut:

$$\theta(h)=\frac{1+2v_F(h)}{1-2v_F(h)} \tag{2.18}$$

2.5 Koefisien Ekstermal

Dalam analisis spasial ekstrem yang perlu diperhatikan adalah ukuran dependensi spasial pada lokasi berdasarkan koefisien ekstermal. Koefisien ekstermal dapat mengukur tingkat dependensi data antara wilayah satu dengan wilayah lainnya. Koefisien ekstermal diperkenalkan oleh Smith yang didefinisikan pada persamaan (2.19) sebagai berikut:

$$\boldsymbol{\theta}\left(h_{j,k}\right) = 2\Phi\left(\frac{\sqrt{h_{j,k}{}^{T} s_{j,k}{}^{-1} h_{j,k}}}{2}\right) \tag{2.19}$$

dimana

$\boldsymbol{\theta}\left(h_{j,k}\right)$ = Nilai koefisien ekstermal

Φ = Fungsi distribusi kumulatif normal standart

$s_{j,k}$ = matriks kovarian dari variabel lokasi ke-*j* dan ke-*k*

$h_{j,k}$ = vektor jarak antara lokasi *j* dengan *k*, perhitunggan jarak berdasarkan jarak euclidean dengan persamaan sebagai berikut $\sqrt{\left(lat_1 - lat_2\right)^2 + \left(lon_1 - lon_2\right)^2}$. Nilai $\boldsymbol{\theta}\left(h_{j,k}\right)$ memiliki kisaran nilai $1 < \boldsymbol{\theta}\left(h_{j,k}\right) < 2$. Nilai $\boldsymbol{\theta}\left(h_{j,k}\right)$ semakin mendekati 1 mengindikasikan bahwa antara dua wilayah memiliki hubungan yang dependen. Nilai $\boldsymbol{\theta}\left(h_{j,k}\right)$ semakin mendekati 2 mengindikasikan bahwa antara dua wilayah memiliki hubungan yang independen (Davidson, Padoan dan Ribatet, 2012)

2.6 Copula

Copula pertama kali diperkenalkan oleh Abe Sklar pada tahun 1959 melalui teorema sklar. Menurut teorema sklar, copula merupakan suatu fungsi yang menghubungkan fungsi distribusi multivariat dengan distribusi marginalnya (Nelsen, 2005). Copula dapat mengeksplorasi dan mengkarakterisasi struktur dependensi antar variabel random melalui fungsi distribusi marginal (Genest, dan Segers, 2010). Copula terbagi menjadi dua macam *families*, yaitu *elliptical* copula dan *archimedian* copula. Untuk kasus *Spatial Extreme Model* copula yang dapat digunakan adalah *elliptical* copula. Copula yang termasuk dalam *elliptical copula* adalah *Gaussian* copula dan *Student's t*-copula (Davidson, Padoan, dan Ribatet, 2012).

2.6.1 Copula Gaussian

Copula Gaussian merupakan copula yang sesuai untuk memberikan model dalam *spatial extreme*. Copula Gaussian atau Copula Normal diperoleh dari transformasi variabel random ke distribusi normal standar. Dalam Copula Gaussian untuk kasus *spatial extreme* proses transformasi menggunakan distribusi marginal GEV dengan persamaan transformasi didefinisikan pada persamaan (2.20) sebagai berikut:

$$u_j = F_{X_j}(x_{ij}) \tag{2.20}$$

dimana

F_{X_j} : CDF dari distribusi GEV

x_{ij} : data observasi ke-*i* stasiun ke-*j*

Menurut Nelsen (2005), jika fungsi distribusi marginal dari u_j kontinu maka u_j adalah copula unik. CDF copula gaussian mengikuti persamaan (2.21) sebagai berikut:

$$C(u_1, u_2 ..., u_m) = \Phi(\Phi^{-1}(u_1), \Phi^{-1}(u_2), ..., \Phi^{-1}(u_m); \rho) \tag{2.21}$$

dengan

Φ : CDF distribusi gaussian

ρ : fungsi korelasi

Fungsi korelasi yang digunakan dalam penelitian ini adalah korelasi *whittle-mattern* didefinisikan pada persamaan berikut (Davidson, 2012):

$$\rho(h) = \left\{2^{(c_0+c)-1}\Gamma(c_0+c)\right\}^{-1}\left(\|h\|/a\right)^{(c_0+c)} K_{(c_0+c)}\left(\|h\|/a\right) \tag{2.22}$$

dengan Γ adalah fungsi Gamma, $K_{(c_0+c)}$ adalah fungsi Bessel dengan derajat (c_0+c), a adalah parameter *range* dan (c_0+c) adalah parameter *sill*. Dari CDF copula gaussian dibentuk pdf copula gaussian, pdf copula gaussian didefinisikan pada persamaan (2.23) sebagai berikut:

$$c(u_1,u_2,...,u_m)=\frac{\partial}{\partial u_1}.\frac{\partial}{\partial u_2}...\frac{\partial}{\partial u_m}.C(u_1,u_2,...,u_m) \tag{2.23}$$

Menurut teorema sklar peluang bersama copula didefinisikan dengan perkalian antara pdf ditribusi marginal dengan fungsi CDF copula, sehingga fungsi peluang bersama didefinisikan pada persamaan (2.24) sebagai berikut:

$$f\left(x_1,x_2,...,x_m\right)=f_{x_1}\left(x_1\right)\cdot...\cdot f_{x_m}\left(x_m\right)\cdot c\left(u_1,...,u_m\right) \tag{2.24}$$

(Schölzel dan Friederichs, 2008)

2.7 *Maximum Pairwise Likelihood Estimation* (MPLE)

Menurut Davidson (2012), Estimasi parameter Copula Gaussian untuk spasial ekstrem dapat menggunakan *Maximum Pairwise Likelihood Estimation* (MPLE). MPLE adalah metode estimasi parameter yang menggunakan fungsi *pairwise*/berpasangan dari dua variabel. Seperti halnya metode MLE, estimasi menggunakan metode ini dilakukan dengan menurunkan satu kali fungsi *ln likelihood* terhadap parameter yang diestimasi dan menyamakannya dengan vektor nol. Metode MPLE menggantikan fungsi ($l(\boldsymbol{\beta})$) pada MLE dengan fungsi *pairwise likelihood* $\ell_p(\boldsymbol{\beta})$ yang didefinisikan pada persamaan (2.25) sebagai berikut:

$$\ell_p\left(\hat{\boldsymbol{\beta}}\right)=\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(f\left(u_{ji},u_{ki};\hat{\boldsymbol{\beta}}\right)\right) \tag{2.25}$$

$f\left(u_{ji},u_{ki};\hat{\boldsymbol{\beta}}\right)$ adalah distribusi bersama copula gaussian dengan parameter $\boldsymbol{\beta}$ dan i=1,2,...,n adalah observasi pada masing-masing variabel. Copula mentrans-formasikan variabel x ke unit margin copula u seperti definisi persamaan (2.20). Estimasi parameter $\boldsymbol{\beta}_{\boldsymbol{\mu}},\boldsymbol{\beta}_{\boldsymbol{\sigma}}$, dan β_ξ, dapat diperoleh jika pembentukan fungsi *likelihood* didasarkan pada $f\left(u_{ji}, u_{ki}; \hat{\boldsymbol{\beta}}\right)$ dengan $\boldsymbol{\beta}_{\boldsymbol{\mu}} = \begin{bmatrix} \beta_{\mu,0} \\ \beta_{\mu,1} \\ \beta_{\mu,2} \end{bmatrix}$, $\boldsymbol{\beta}_{\boldsymbol{\sigma}} = \begin{bmatrix} \beta_{\sigma,0} \\ \beta_{\sigma,1} \\ \beta_{\sigma,2} \end{bmatrix}$ dan β_{ξ}.

2.7.1 *Confidence Interval*

Setelah mendapatkan estimasi parameter GEV copula, selanjutnya mencari *confidence interval* sebagai batas bawah dan batas atas dari estimasi. Masing-masing parameter yang terdapat dalam distribusi GEV copula dihitung interval konfidensinya menggunakan pendekatan standart normal baku. *Confidence Interval* dengan tingkat kepercayaan 100(1-α)% untuk estimasi parameter $\boldsymbol{\beta}_{\boldsymbol{\mu}}, \boldsymbol{\beta}_{\boldsymbol{\sigma}}$, dan β_{ξ} sebagai berikut:

$$\hat{\beta}_{\mu,0} - z_{\alpha/2}\mathrm{SE}\left(\hat{\beta}_{\mu,0}\right) < \beta_{\mu,0} < \hat{\beta}_{\mu,0} + z_{\alpha/2}\mathrm{SE}\left(\hat{\beta}_{\mu,0}\right) \tag{2.26}$$

$$\hat{\beta}_{\mu,1} - z_{\alpha/2}\mathrm{SE}\left(\hat{\beta}_{\mu,1}\right) < \beta_{\mu,1} < \hat{\beta}_{\mu,1} + z_{\alpha/2}\mathrm{SE}\left(\hat{\beta}_{\mu,1}\right) \tag{2.27}$$

$$\hat{\beta}_{\sigma,1} - z_{\alpha/2}\mathrm{SE}\left(\hat{\beta}_{\sigma,1}\right) < \beta_{\sigma,1} < \hat{\beta}_{\sigma,1} + z_{\alpha/2}\mathrm{SE}\left(\hat{\beta}_{\sigma,1}\right) \tag{2.28}$$

$$\hat{\beta}_{\sigma,2} - z_{\alpha/2}\mathrm{SE}\left(\hat{\beta}_{\sigma,2}\right) < \beta_{\sigma,2} < \hat{\beta}_{\sigma,2} + z_{\alpha/2}\mathrm{SE}\left(\hat{\beta}_{\sigma,2}\right) \tag{2.29}$$

$$\hat{\beta}_{\xi} - z_{\alpha/2}\mathrm{SE}\left(\hat{\beta}_{\xi}\right) < \beta_{\xi} < \hat{\beta}_{\xi} + z_{\alpha/2}\mathrm{SE}\left(\hat{\beta}_{\xi}\right) \tag{2.30}$$

Dengan SE adalah *standart error* dari masing-masing parameter (Herrhyanto, 2003).

2.8 Max-Stable Process

Dalam konsep spasial ekstrem terdapat dua metode pendekatan yaitu *max-stable* dan *copula* (Davidson, 2012). Perbedaan dari dua metode ini adalah pada saat memodelkan dan proses transformasinya. Untuk Pemodelan dan estimasi, Copula menggunakan model Copula elliptical yaitu *gaussian* dan *student t*, sementara *max-stable* menggunakan model schlater, smith dan brown-resnick.

Untuk proses transformasinya kedua pendekatan ini menggunakan proses sama yaitu *max-stabel* karena proses *max-stabel* membawa data ke distribusi frechet, akan tetapi proses transformasi copula menggunakan transformasi sifat ke-1 dan proses *max-stabel* menggunakan transformasi sifat ke-2. Sifat transformasi max-stable adalah sebagai berikut:

1. Distribusi marginal satu dimensionalnya mengikuti distribusi GEV

   $X \sim GEV\left(\mu,\lambda,\xi\right)$ dengan fungsi distribusi sebagai berikut:

$$F\left(\mu,\sigma,\xi\right)=\exp\left[-\left\{1+\frac{\xi\left(x-\mu\right)}{\sigma}\right\}^{-1/\xi}\right],-\infty<\mu,\xi<\infty,\sigma>0 \qquad (2.31)$$

   dimana

   μ = parameter lokasi

   σ = parameter skala (*scale*)

   ξ = parameter bentuk (*shape*)

2. Distribusi marginal k-dimensionalnya mengikuti distribusi *multivariate extreme value*.

   $\left\{Z\left(j\right)\right\}$ adalah proses *max-stable* yang memiliki margin Fréchet unit dengan fungsi distribusi $F(z)=\exp\left(-1/z\right),\quad z>0$. Proses ini dapat diperoleh dengan mentransformasi $\left\{x\left(j\right)\right\}$ menjadi persamaan (2.32).

$$\left\{Z\left(j\right)\right\}\equiv\left[\left\{1+\frac{\xi\left(x-\mu\right)}{\sigma}\right\}^{1/\xi}\right] \qquad (2.32)$$

   dimana $\mu\left(x\right),\xi\left(x\right),\sigma\left(x\right)$ adalah suatu fungsi kontinyu. Proses Z ini juga disebut proses *max-stable* (Padoan, Ribatet, Sisson, 2010).

2.9 Pemilihan Model Terbaik

Akaike Information Criterion (AIC) digunakan untuk memilih model *trend surface* terbaik, dengan model sebagai berikut:

$$\begin{aligned}
\hat{\mu}(j) &= \widehat{\beta}_{\mu,0} + \widehat{\beta}_{\mu,1}\, longitude(j) + \widehat{\beta}_{\mu,2}\, latitude(j) \\
\hat{\sigma}(j) &= \widehat{\beta}_{\sigma,0} + \widehat{\beta}_{\sigma,1}\, longitude(j) + \widehat{\beta}_{\sigma,2}\, latitude(j) \\
\hat{\xi}(j) &= \widehat{\beta}_{\xi,0}
\end{aligned} \tag{2.33}$$

Kriteria pemilihan model memiliki peran penting dalam menentukan model yang terbaik. Pada beberapa konteks tertentu, memilih model yang sederhana lebih baik daripada model yang kompleks. Menurut Ligas dan Banasick (2012) *Akaike Information Criterion* (AIC) didefinisikan dengan persamaan (2.34) sebagai berikut:

$$AIC = -2\ell_p\left(\widehat{\boldsymbol{\beta}}\right) + 2q \tag{2.34}$$

dimana $\ell_p\left(\widehat{\boldsymbol{\beta}}\right)$ adalah fungsi *ln pairwise likelihood* didefinisikan pada persamaan (2.35) sebagai berikut:

$\ell_p\left(\widehat{\boldsymbol{\beta}}\right)$ = fungsi *ln pairwise likelihood*

$$\ell_p\left(\widehat{\boldsymbol{\beta}}\right) = \sum_{i=1}^{k}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \ln\left(f\left(u_{ji}, u_{ki}; \widehat{\boldsymbol{\beta}}\right)\right) \tag{2.35}$$

dengan i=1,2,…,n, j=1,2,…,m-1, k=2,3,...,m dan q adalah banyaknya parameter yang ditaksir. Nilai AIC yang lebih rendah menunjukkan model yang lebih baik.

2.10 Return Level

Dalam *spatial extreme* hal yang menarik bukan hanya pada penaksiran parameter akan tetapi juga dapat menentukan *return level*. *Return level* adalah nilai maksimum pada periode mendatang. Konsep *return level* dan periode ulang biasanya digunakan untuk menyampaikan informasi tentang kemungkinan peristiwa langka seperti curah hujan yang ekstrem sehingga akan berdampak banjir. *Return level* pada lokasi (j) tertentu disimbolkan $Z_P\left(j\right)$ dengan proses perhitungan *return level* didefinisikan pada persamaan (2.36) sebagai berikut:

$$Z_P\left(j\right) = \widehat{\mu}\left(j\right) - \frac{\widehat{\sigma}\left(j\right)}{\widehat{\xi}\left(j\right)}\left(1 - \left(\ln(1 - \frac{1}{T}\right)^{-\widehat{\xi}(j)}\right) \tag{2.36}$$

dimana

$\hat{\mu}$: parameter lokasi

$\hat{\sigma}$: parameter skala (*scale*)

$\hat{\xi}$: parameter bentuk (*shape*)

T : periode blok

(Gilli dan Kellezi, 2006)

2.11 *Root Mean Square Error* (RMSE)

Root Mean Square Error (RMSE) digunakan untuk mengetahui apakah estimasi parameter mempunyai kinerja yang baik dan layak untuk digunakan. Pengukuran RMSE dilakukan dengan memperhatikan selisih nilai estimasi dan nilai aktual yang diperoleh dari data *testing.* RMSE didefinisikan pada persamaan (2.37) sebagai berikut:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{J}(x_i - \hat{x}_i)^2}{J}} \tag{2.37}$$

dengan J merupakan banyaknya lokasi, x_i merupakan nilai observasi aktual yang didapat dari data testing dan $\hat{x}_i$ merupakan nilai dugaan atau prediksi pada periode ulang (T) (Chai dan Draxler, 2014).

2.12 Curah Hujan Ekstrem

Curah hujan dapat diartikan sebagai ketinggian air yang tekumpul dalam tempat yang datar, tidak menguap, tidak meresap, dan tidak mengalir. Untuk mengukur curah hujan, digunakan alat yang disebut *Observarium* dan umumnya curah hujan dinyatakan dalam milimeter. Curah hujan satu milimeter artinya pada luasan satu meter persegi dalam tempat yang datar tertampung air setinggi satu milimeter atau tertampung air sebanyak satu liter. Sifat curah hujan adalah perbandingan antara jumlah curah hujan selama rentang waktu yang ditetapkan (rata-rata selama 1990-2015). Sifat hujan dibagi menjadi 3 (tiga) katagori antara lain:

1. Di atas normal (AN) terjadi jika nilai curah hujan lebih dari 115% terhadap rata-ratanya.
2. Normal (N) terjadi jika nilai curah hujan antara 85% sampai 115% terhadap rata-ratanya.
3. Di bawah normal (BN) jika curah hujan kurang dari 85% (BMKG, 2016).

Selain itu curah hujan juga dibedakan menjadi tiga jika ditinjau besarnya intensitasnya yang meliputi:

1. Curah hujan rendah (150-200 mm/bulan)
2. Curah hujan sedang (200-250 mm/bulan)
3. Curah hujan tinggi (250-300 mm/bulan)

Menurut BMKG dalam Kadarsah (2007), berdasarkan distribusi data rata-rata curah hujan bulanan, curah hujan di Indonesia dibedakan menjadi tiga tipe, yaitu :

1. Tipe Ekuatorial

Pola ekuatorial dicirikan oleh tipe curah hujan dengan bentuk bimodal (dua puncak musim hujan) yang biasanya terjadi sekitar bulan Maret dan Oktober atau pada saat terjadi ekuinoks, yaitu waktu atau peristiwa matahari berada dalam bidang katulistiwa bumi dimana peristiwa ini terjadi dua kali dalam setahun. Di Indonesia, curah hujan yang mengikuti pola ini terjadi di sebagian besar wilayah Sumatra dan Kalimantan.

2. Tipe Monsoon

Curah hujan dipengaruhi oleh tiupan angin monsoon dan bersifat unimodal (satu puncak musim hujan, DJF (Desember-Januari-Februari) musim hujan, JJA (Juni-Juli-Agustus) musim kemarau. Tipe hujan ini terjadi di wilayah Indonesia bagian selatan, seperti di ujung Pulau Sumatra bagian selatan, Jawa, Bali, Nusa Tenggara dan Maluku selatan.

3. Tipe Lokal

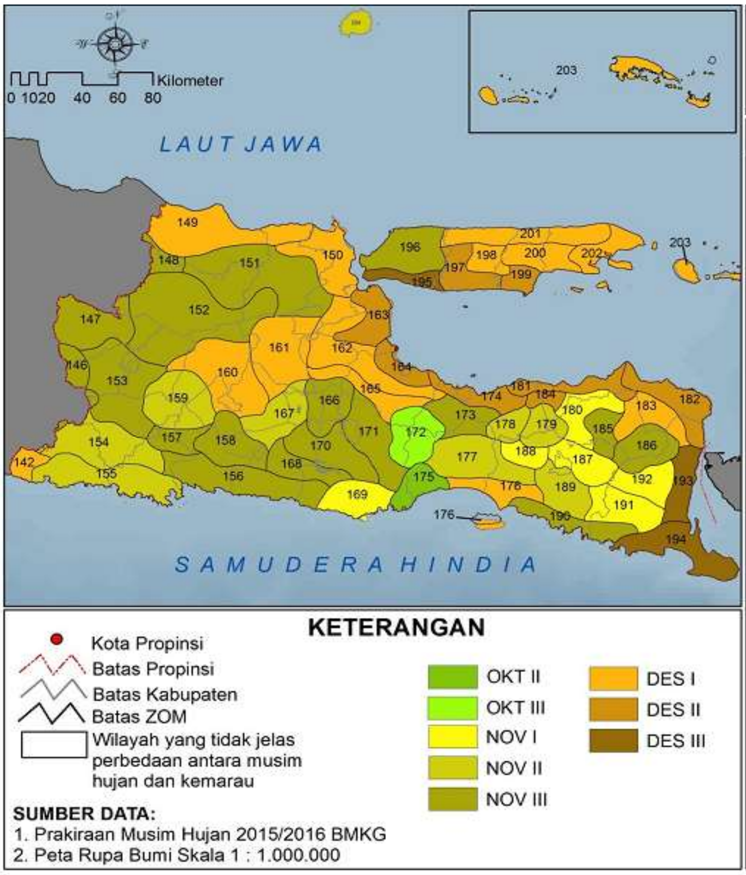
Curah hujan dipengaruhi oleh kondisi lingkungan setempat, yakni adanya perairan sebagai sumber penguapan dan pegunungan sebagai daerah tangkapan hujan. Pola curah hujan lokal memiliki distribusi hujan bulanan kebalikan dengan pola monsoon, dicirikan oleh bentuk pola hujan unimodal (satu puncak hujan), tetapi bentuknya berlawanan dengan tipe hujan monsun.

Curah hujan dengan intensitas lebih dari 50 milimeter per hari menjadi parameter terjadinya hujan dengan intensitas lebat, sedangkan curah hujan ekstrem memiliki curah hujan lebih dari 100 milimeter per hari. (BMKG, 2016).

2.13 Zona Musim

Zona Musim (ZOM) adalah daerah yang pola hujan rata-ratanya memiliki perbedaan yang jelas antara periode musim kemarau dan musim hujan. Daerah-daerah yang pola hujan rata-ratanya tidak memiliki perbedaan yang jelas antara periode musim kemarau dan musim hujan disebut, non ZOM. Luas suatu wilayah ZOM tidak selalu sama dengan luas suatu wilayah administrasi pemerintahan. Dengan demikian, suatu wilayah ZOM bisa terdiri dari beberapa kabupaten, dan sebaliknya suatu wilayah kabupaten bisa terdiri atas beberapa ZOM

Zona musim merupakan pembagian daerah-daerah di Indonesia berdasarkan pola distribusi curah hujan rata-rata bulanan. Berdasarkan hasil analisis data periode 30 tahun terakhir (1981-2010), secara klimatologis wilayah Indonesia terdapat 407 pola iklim. Dimana 342 pola merupakan Zona Musim, Sedangkan 65 pola lainnya adalah Non Zona Musim (Non ZOM) (BMKG, 2016). Berdasarkan Gambar 2.5, kabupaten Ngawi berada di Zona Musim 146 (Karanganyar bagian timur, wonogiri bagian timur laut, magetan bagian barat, Ngawi bagian selatan) dan Zona Musim 147 (Grobogan bagian selatan, Sragen bagian utara, Ngawi dan Bojonegoro bagian barat daya).



Sumber: BMKG, 2016

Gambar 2.5 Pembagian Zona Musim (ZOM) di Provinsi Jawa Timur

(*Halaman ini sengaja dikosongkan*)

BAB 3
METODE PENELITIAN

3.1 Sumber Data

Data yang digunakan dalam penelitian ini merupakan data sekunder yang bersumber dari Badan Meteorologi, Klimatologi dan Geofisika berupa data curah hujan harian di 11 pos pengukuran di Kabupaten Ngawi tahun 1990-2015. Data ini merupakan data penelitian tim dosen Laboratorium Lingkungan dan Kesehatan Statistika ITS

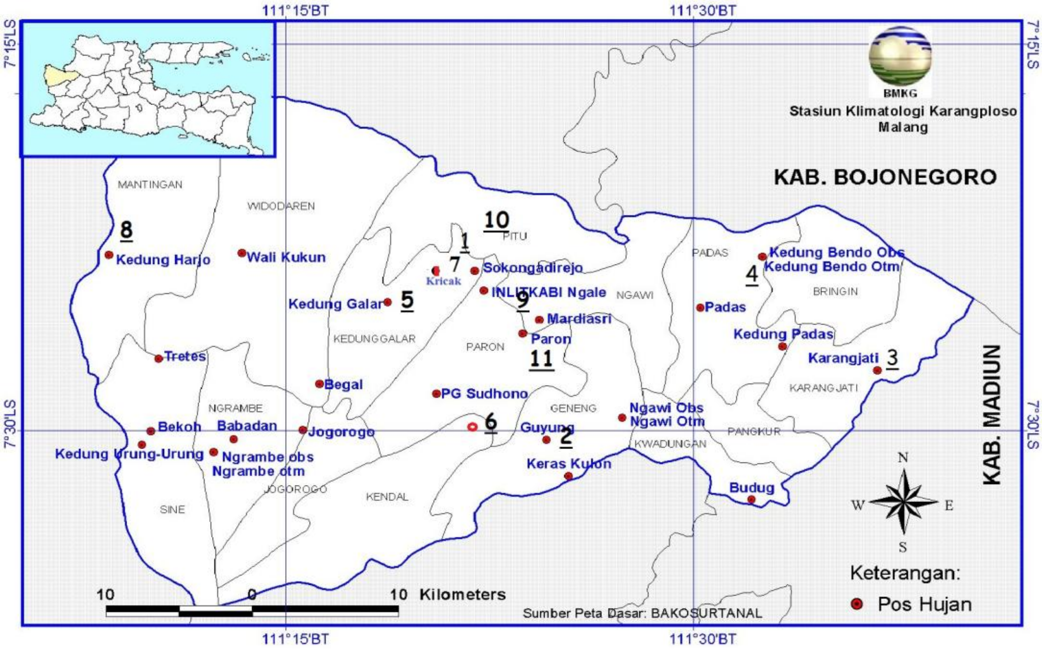
3.2 Variabel Penelitian

Variabel yang digunakan dalam penelitian adalah curah hujan yang diambil dari 11 pos hujan di Kabupaten Ngawi. Sebelas Pos hujan yang diamati disajikan pada Tabel 3.1 sebagai berikut:

Tabel 3.1 Koordinat 11 Pos hujan Kabupaten Ngawi

No	Pos Hujan	Longitude (*u*)	Latitude (*v*)
1	Gemarang	111,366	-7,396
2	Guyung	111,410	-7,505
3	Karangjati	111,613	-7,461
4	Kedungbendo	111,542	-7,386
5	Kedunggalar	111,312	-7,408
6	Kendal	111,288	-7,560
7	Kricak	111,344	-7,394
8	Mantingan	111,149	-7,385
9	Mardisari	111,405	-7,428
10	Papungan	111,369	-7,383
11	Paron	111,395	-7,437

Gambar 3.1 menggambarkan peta persebaran dan letak seluruh Pos hujan yang ada di kabupaten ngawi. Kabupaten Ngawi terbagi dalam dua zona musim yaitu zona musim 146 dan zona musim 147. Pos hujan yang akan digunakan adalah seluruh Kabupaten Ngawi yang berada pada Zona musim 147, kecuali stasiun yang terletak pada daerah ngawi bagian selatan karena terletak pada zona iklim 146.



Gambar 3.1 Peta Pos Hujan Kabupaten Ngawi

Data dibagi menjadi dua, yaitu data *training* untuk analisis dan data *testing* untuk validasi model. Data curah hujan harian tahun 1990-2010 digunakan sebagai data *training*, sedangkan untuk validasi digunakan data tahun 2010-2015. Struktur data yang digunakan ditunjukkan pada Tabel 3.1

Tabel 3.1 Struktur Data Penelitian

No	Harian	Bulan	Tahun	Pos Hujan 1	Pos Hujan 2		Pos Hujan 11
1	1	12	1990	$y_{1,1}$	$y_{1,2}$	...	$y_{1,11}$
2	2	12	1990	$y_{2,1}$	$y_{2,2}$	...	$y_{2,11}$
3	3	12	1990	$y_{3,1}$	$y_{3,2}$	...	$y_{3,11}$
⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮
19	19	12	1990	$y_{19,1}$	$y_{19,2}$	...	$y_{19,11}$
20	20	12	1990	$y_{20,1}$	$y_{20,2}$	...	$y_{20,11}$
21	21	12	1990	$y_{21,1}$	$y_{21,2}$	...	$y_{21,11}$
⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮
9131	31	8	2015	$Y_{9131,1}$	$Y_{9131,2}$	...	$Y_{9131,11}$

3.3 Tahapan Penelitian

Tahapan penelitian yang dilakukan untuk mencapai dua tujuan penelitian adalah :

A. Estimasi parameter $\boldsymbol{\beta}_{\mu}, \boldsymbol{\beta}_{\sigma,}$ dan β_{ξ} pada model *trend surface* Copula Gaussian menggunakan metode MPLE.

1. Menyusun pdf Copula Gaussian dari CDF Copula Gaussian.

   CDF Copula Gaussian didefinisikan pada persamaan sebagai berikut:

   $$C(u_1, u_2 \dots, u_m) = \Phi(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \dots, \Phi^{-1}(u_m); \rho)$$

   pdf Copula Gaussian diperoleh dari turunan CDF Copula Gaussian sebagai berikut:

   $$\begin{aligned} c(u_1, \dots, u_m) &= \frac{\partial^m}{\partial u_1 \dots \partial u_m} C(u_1, \dots, u_m) \\ &= \frac{\partial^m}{\partial u_1 \dots \partial u_m} \Phi\left(\Phi^{-1}[u_1], \dots, \Phi^{-1}[u_m]\right) \end{aligned}$$

2. Menyusun fungsi *pairwise likelihood* dari pdf Copula Gaussian.

   Fungsi distribusi bersama Copula Gaussian diperoleh dari perkalian antara pdf ditribusi marginal dengan fungsi CDF copula didefinisikan sebagai persamaan berikut:

   $$f(x_1, x_2, \dots, x_m) = f_{x_1}(x_1) \cdot \dots \cdot f_{x_m}(x_m) \cdot c(u_1, \dots, u_m)$$

   dari Fungsi distribusi bersama dapat dibentuk fungsi *pairwise likelihood* dengan persamaan sebagai berikut:

   $$\begin{aligned} L_p(\widehat{\boldsymbol{\beta}}) &= \prod_{i=1}^{n} \prod_{j=1}^{m-1} \prod_{k=j+1}^{m} f(u_{ji}, u_{ki}; \widehat{\boldsymbol{\beta}}) \\ &= \prod_{i=1}^{n} \prod_{j=1}^{m-1} \prod_{k=j+1}^{m} \Big(f_{x_j}(x_{ji}) \cdot f_{x_k}(x_{ki}) . \\ &\quad \exp\left(\frac{1}{2} \left[\Phi^{-1}(u_{ji}) \ \Phi^{-1}(u_{ki}) \right]^T . \left(\rho(h)^{-1} \right) . \left[\Phi^{-1}(u_{ji}) \ \Phi^{-1}(u_{ki}) \right] \right) \cdot |\rho(h)|^{-0.5} \Big) \end{aligned}$$

3. Menyusun Fungsi *ln pairwise likelihood*.

   Dari Fungsi *pairwise likelihood* dibentuk fungsi *ln pairwise likelihood* sehingga diperoleh persamaan berikut:

$$\ell_p\left(\widehat{\boldsymbol{\beta}}\right)=\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(f_{x_j}\left(x_{ji}\right)\cdot f_{x_k}\left(x_{ki}\right)\right)\cdot\left(\frac{1}{2}\left[\Phi^{-1}\left(u_{ji}\right)\ \Phi^{-1}\left(u_{ki}\right)\right]^T\cdot\left(\rho\left(h\right)^{-1}\right)\cdot\right.$$
$$\left.\left[\Phi^{-1}\left(u_{ji}\right)\ \Phi^{-1}\left(u_{ki}\right)\right]\right)\text{-}0.5\ln|\rho(h)|$$

4. Melakukan penurunan pertama parameter $\boldsymbol{\beta}_{\boldsymbol{\mu}},\boldsymbol{\beta}_{\boldsymbol{\sigma}},\text{ dan }\beta_{\xi}$ terhadap fungsi *ln pairwise likelihood* dan menyamadenganka‌n dengan vektor nol.

$$\frac{\partial\ell_p\left(\widehat{\boldsymbol{\beta}}\right)}{\partial\boldsymbol{\beta}_{\boldsymbol{\mu}}}=\frac{\partial\left(\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(f_{x_j}\left(x_{ji}\right)\cdot f_{x_k}\left(x_{ki}\right)\right)\cdot\left(\frac{1}{2}\left[\Phi^{-1}\left(u_{ji}\right)\ \Phi^{-1}\left(u_{ki}\right)\right]^T\cdot\left(\rho\left(h\right)^{-1}\right)\cdot *\right.\right.}{\partial\boldsymbol{\beta}_{\boldsymbol{\mu}}}$$
$$*=\left.\left[\Phi^{-1}\left(u_{ji}\right)\ \Phi^{-1}\left(u_{ki}\right)\right]\right)\text{-}0.5\ln|\rho(h)|\Big)$$

$$\frac{\partial\ell_p\left(\widehat{\boldsymbol{\beta}}\right)}{\partial\boldsymbol{\beta}_{\boldsymbol{\sigma}}}=\frac{\partial\left(\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(f_{x_j}\left(x_{ji}\right)\cdot f_{x_k}\left(x_{ki}\right)\right)\cdot\left(\frac{1}{2}\left[\Phi^{-1}\left(u_{ji}\right)\ \Phi^{-1}\left(u_{ki}\right)\right]^T\cdot\left(\rho\left(h\right)^{-1}\right)\cdot *\right.\right.}{\partial\boldsymbol{\beta}_{\boldsymbol{\sigma}}}$$
$$*=\left.\left[\Phi^{-1}\left(u_{ji}\right)\ \Phi^{-1}\left(u_{ki}\right)\right]\right)\text{-}0.5\ln|\rho(h)|\Big)$$

$$\frac{\partial\ell_p\left(\widehat{\boldsymbol{\beta}}\right)}{\partial\boldsymbol{\beta}_{\xi}}=\frac{\partial\left(\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(f_{x_j}\left(x_{ji}\right)\cdot f_{x_k}\left(x_{ki}\right)\right)\cdot\left(\frac{1}{2}\left[\Phi^{-1}\left(u_{ji}\right)\ \Phi^{-1}\left(u_{ki}\right)\right]^T\cdot\left(\rho\left(h\right)^{-1}\right)\cdot *\right.\right.}{\partial\boldsymbol{\beta}_{\xi}}$$
$$*=\left.\left[\Phi^{-1}\left(u_{ji}\right)\ \Phi^{-1}\left(u_{ki}\right)\right]\right)\text{-}0.5\ln|\rho(h)|\Big)$$

5. Hasil estimasi tidak close form sehingga digunakan pendekatan secara numerik dengan menggunakan analisis numerik yaitu Nelder-Mead.

B Prosedur pemodelan *Spatial Extreme Value* dengan pendekatan *Copula* terhadap data curah hujan ekstrem di Kabupaten ngawi

1. Menghimpun data curah hujan dari 11 pos hujan di Kabupaten Ngawi tahun 1990-2015.
2. Melakukan analisis deskriptif data untuk *mean, max, min,* kurtosis dan *skewness*.

3. Mengidentifikasi distribusi data curah hujan di masing-masing Pos hujan untuk mengetahui adanya distribusi data *heavy tail* dan nilai ekstrem dengan histogram.
4. Mengambil sampel ekstrem dengan metode *Block Maxima*, dengan membuat blok periode waktu tiga bulan yaitu Desember-Januari-Februari (DJF), Maret-April-Mei (MAM), Juni-Juli-Agustus (JJA), dan September-Oktober-Nopember (SON) untuk data curah hujan 1980-2015. Sampel nilai ekstrem diambil dari nilai maksimum curah hujan dari masing-masing blok.
5. Membagi data menjadi data *training* dan data *testing*. Data *training* merupakan data yang dianalisis untuk membentuk model, sedangkan data *testing* digunakan untuk validasi model yang diperoleh. Data training dari bulan Desember tahun 1990 sampai bulan Agustus tahun 2010. Data *testing* dari dari bulan September tahun 2010 sampai bulan Nopember tahun 2015
6. Menguji kesesuaian distribusi *generalized extreme value* (GEV) setiap lokasi dengan uji Anderson Darling.
7. Menentukan estimasi parameter untuk $\hat{\mu}, \hat{\sigma}$, dan $\hat{\xi}$ *univariat* pada masing-masing lokasi/pos dengan MLE dan diselesaikan secara numerik dengan metode iterasi Nelder-Mead.
8. Menghitung dependensi spasial data curah hujan dengan menggunakan koefisien ekstremal.
9. Melakukan transformasi setiap veriabel random ke margin *copula*
10. Melakukan estimasi parameter distribusi GEV spasial dengan pendekatan *copula* untuk data curah hujan ekstrem.
11. Memilih model *trend surface* terbaik dari semua kombinasi model melalui nilai AIC terkecil.
12. Menentukan *Confidence Interval* untuk parameter distribusi GEV spasial dengan pendekatan copula untuk model yang terbaik.
13. Menentukan nilai *return level* dari data curah hujan di Kabupaten Ngawi.

(*Halaman ini sengaja dikosongkan*)

BAB 4

HASIL DAN PEMBAHASAN

Bab ini membahas estimasi parameter distribusi *generalized extreme value* (GEV) pada model *spatial extreme value* dengan pendekatan copula, metode yang digunakan untuk melakukan estimasi adalah *maximum pairwise likelihood estimation* (MPLE). Copula yang digunakan dalam penelitian ini adalah copula gaussian. Selanjutnya menerapkan pemodelan *spatial* ekstrem pada data curah hujan di sebelas Pos hujan di Kabupaten Ngawi. Pemodelan spatial ekstrem diawali dengan pra-pemrosesan data dan deskripsi data untuk mengetahui gambaran umum karakteristik curah hujan di Kabupaten Ngawi. Kemudian dibahas pula pengambilan sampel ekstrem dengan metode *block maxima*. Pada bagian akhir dibahas dependensi *spatial*, estimasi parameter, dan menentukan *return level* pada masing-masing pos hujan.

4.1 Estimasi Parameter Distribusi GEV *Spatial* Model Copula Gaussian dengan metode *Maximum Pairwise Likelihood Estimation* (MPLE)

Metode Esimasi Parameter untuk GEV *Spatial* Model copula gaussian menggunakan *maximum pairwise likelihood estimation* (MPLE), hal ini dikarenakan dalam konsep *spatial* dilihat jarak dari pasangan 2 lokasi yang berbeda oleh karena itu digunakan *pairwise* dalam menentukan estimasi *spatial* GEV.

Dalam penelitian ini pendekatan *spatial* yang digunakan adalah copula. Copula lebih tepat digunakan untuk data yang bersifat *heavytail*. Dari tiga macam distribusi GEV yaitu *reversed* weibull, gumbel, dan frechet, distribusi yang bersifat paling *heavytail* adalah distribusi frechet. Transformasi GEV ke unit margin frechet merupakan suatu proses *max-stable*. Transformasi parameter dari GEV *univariat* ke unit *marginal* frechet diperlukan apabila hasil dari estimasi parameter GEV tidak berbentuk distribusi frechet, dimana parameter ξ untuk distribusi frechet adalah $\xi > 0$. Setelah parameter GEV berbentuk distribusi frechet selanjutnya dilakukakan transformasi lagi ke copula dengan persamaan transfomasi sebagai pada persamaan (4.1) sebagai berikut:

$$u_j = F_j\left(x_{ij}\right) \qquad (4.1)$$

dimana u adalah hasil transformasi copula dan F_j merupakan CDF dari distribusi GEV mengikuti persamaan (2.1), F_j ini merupakan suatu proses *max-stable* yang memiliki unit *margin* frechet dengan fungsi distribusinya seperti persamaan (4.2) berikut:

$$F_j = \exp\left(-\frac{1}{z}\right) \qquad (4.2)$$

dimana z merupakan suatu proses *max-stable* yang mentransformasi data ke unit *margin* frechet dengan persamaan z seperti persamaan (4.3) sebagai berikut:

$$z = \left(1 + \frac{\hat{\xi}\left(x_{ij} - \hat{\mu}\right)}{\hat{\sigma}}\right)^{\frac{1}{\hat{\xi}}} \qquad (4.3)$$

dimana

x_{ij} adalah nilai ekstrem observasi ke-*i* stasiun ke-*j*

$\hat{\mu}$ adalah parameter lokasi (*location*)

$\hat{\sigma}$ adalah parameter skala (*scale*)

$\hat{\xi}$ adalah parameter bentuk (*shape*)

Parameter $\hat{\mu}$, $\hat{\sigma}$, dan $\hat{\xi}$ adalah parameter yang diperoleh dari hasil esrimasi parameter GEV secara *univariat*.

Setelah melakukan proses transformasi ke copula, dilakukan proses estimasi parameter copula GEV *spatial*. Parameter yang akan diestimasi adalah parameter $\boldsymbol{\beta}_{\boldsymbol{\mu}}$, $\boldsymbol{\beta}_{\boldsymbol{\sigma}}$, dan β_{ξ}. Copula yang digunakan adalah copula gaussian karena copula gaussian membawa distribusi *multivariate extreme value* ke dimensi tak hingga (*infinite dimensional*). Pada tinjauan pustaka BAB 2 dijelaskan bahwa copula gaussian yang memiliki *Cumulative Distribution Function* (CDF) yang didefinisikan pada persamaan (4.3) sebagai berikut:

$$C(u_1,\ldots,u_m) = \Phi\left(\Phi^{-1}[u_1],\ldots,\Phi^{-1}[u_m]\right) \tag{4.3}$$

untuk mengestimasi parameter copula gaussian menggunakan metode MPLE maka perlu menyusun pdf copula gaussian dari CDF copula gaussian, Fungsi pdf copula gaussian diperoleh dari turunan fungsi CDF copula gaussian yang didefinisikan dalam persamaan (4.4) sebagai berikut:

$$\begin{aligned} c(u_1,\ldots,u_m) &= \frac{\partial^m}{\partial u_1 \ldots \partial u_m} C(u_1,\ldots,u_m) \\ &= \frac{\partial^m}{\partial u_1 \ldots \partial u_m} \Phi\left(\Phi^{-1}[u_1],\ldots,\Phi^{-1}[u_m]\right) \end{aligned} \tag{4.4}$$

Φ adalah fungsi distribusi kumulatif multivariat dengan korelasi ρ , Φ^{-1} adalah invers CDF distribusi normal. Sehingga fungsi distribusi copula sebagai berikut:

$$c(\mathbf{u}) = |\left(\rho(h)\right)|^{-1/2} \exp\left[\mathbf{u}^T \cdot \rho(h)^{-1} \cdot \mathbf{u}\right]$$

dimana h adalah jarak lokasi 1 dan lokasi 2 (antar lokasi), $\mathbf{u}$ adalah transformasi copula, dan $\rho(h)$ adalah fungsi korelasi, dimana korelasi yang digunakan adalah korelasi *whittle-matern* dengan persamaan korelasi didefinisikan pada persamaan (2.22). Dari persamaan (4.4) diperoleh pdf copula gaussian sebagai berikut:

$$\begin{aligned} c(u_1,\ldots,u_m) = \exp&\left(\frac{1}{2}\left(\Phi^{-1}[u_1],\ldots,\Phi^{-1}[u_m]\right)^T . \left(\rho(h)\right)^{-1} \cdot \left(\Phi^{-1}[u_1],\ldots,\Phi^{-1}[u_m]\right)\right) \\ &. |\left(\rho(h)\right)|^{-0.5} \end{aligned}$$

Misalkan $\mathbf{v} = \left(\Phi^{-1}[u_1],\ldots,\Phi^{-1}[u_m]\right)$ maka

$$c(u_1,\ldots,u_m) = \exp\left(\frac{1}{2}\left(\mathbf{v}^T . \left(\rho(h)\right)^{-1} . \mathbf{v}\right)\right) \cdot |\rho(h)|^{-0.5}$$

Setelah mendapatkan fungsi pdf copula gaussian langkah selanjutnya adalah menyusun fungsi peluang bersama dituliskan dalam bentuk copula, fungsi peluang bersama copula disajikan pada persamaan (4.6) sebagai berikut:

$$f(x_1, x_2, ..., x_m) = f_{x_1}(x_1) \cdot ... \cdot f_{x_m}(x_m) \cdot c(u_1, ..., u_m) \tag{4.6}$$

$$= f_{x_1}(x_1) \cdot ... \cdot f_{x_m}(x_m) \cdot \exp\left(\frac{1}{2}\left(\mathbf{v}^T.(\rho(h))^{-1}.\mathbf{v}\right)\right) \cdot |\rho(h)|^{-0.5}$$

dimana f merupakan pdf distribusi GEV karena fungsi *marginal* copula *spatial extreme* menggunakan fungsi *marginal* GEV, pdf dari distribusi GEV mengikuti persamaan (2.2). Dari pdf tersebut, fungsi peluang bersama f dituliskan dalam bentuk bivariat yang disajikan pada persamaan (4.7) sebagai berikut:

$$\begin{aligned} f(x_1, x_2) &= f_{x_1}(x_1) . f_{x_2}(x_2) . c(u_1, u_2) \\ &= f_{x_1}(x_1) . f_{x_2}(x_2) . \exp\left(\frac{1}{2}\left(\mathbf{v}^T.(\rho(h))^{-1}.\mathbf{v}\right)\right) \cdot |\rho(h)|^{-0.5} \\ &= f_{x_1}(x_1) . f_{x_2}(x_2) . \exp\left(\frac{1}{2}\left(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2]\right)^T .(\rho(h))^{-1}\right. \\ &\quad \left.\cdot\left(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2]\right)\right) . |\rho(h)|^{-0.5} \end{aligned} \tag{4.7}$$

dari fungsi peluang bersama dalam bentuk bivariat tersebut, kemudian dilakukan penyusunan fungsi *pairwise likelihood* copula gaussian yang disajikan pada persamaan (4.8) sebagai berikut:

$$\begin{aligned} L_p(\widehat{\boldsymbol{\beta}}) &= \prod_{i=1}^{n} \prod_{j=1}^{m-1} \prod_{k=j+1}^{m} f(u_{ji}, u_{ki}; \widehat{\boldsymbol{\beta}}) \\ &= \prod_{i=1}^{n} \prod_{j=1}^{m-1} \prod_{k=j+1}^{m} \left(f_j(x_{ji}) \cdot f_k(x_{ki}) . \exp\left(\frac{1}{2}\left(\mathbf{v}^T.(\rho(h))^{-1}.\mathbf{v}\right)\right) \cdot |\rho(h)|^{-0.5} \right) \\ &= \prod_{i=1}^{n} \prod_{j=1}^{m-1} \prod_{k=j+1}^{m} \left(f_{x_j}(x_{ji}) \cdot f_{x_k}(x_{ki}) . \right. \\ &\quad \left. \exp\left(\frac{1}{2}\left(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2]\right)^T \cdot (\rho(h))^{-1} \cdot \left(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2]\right)\right) . |\rho(h)|^{-0.5} \right) \\ &= \prod_{i=1}^{n} \prod_{j=1}^{m-1} \prod_{k=j+1}^{m} \left(f_{x_j}(x_{ji}) \cdot f_{x_k}(x_{ki}) \cdot \right. \\ &\quad \exp \sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \left(\frac{1}{2}\left(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2]\right)^T .(\rho(h))^{-1} \cdot \left(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2]\right)\right) \\ &\quad \left. \cdot \rho(h)|^{-0.5} \right) \end{aligned} \tag{4.8}$$

Setelah membentuk fungsi *pairwise likelihood* langkah selanjutnya adalah menyusun fungsi *ln pairwise likelihood*. Persamaan (4.9) merupakan fungsi *ln pairwise likelihood* dari copula gaussian.

$$\ell_p\left(\hat{\boldsymbol{\beta}}\right)=\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(f_{x_j}\left(x_{ji}\right)\cdot f_{x_k}\left(x_{ki}\right)\right)\cdot\left(\frac{1}{2}\left(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2]\right)^T.\left(\rho(h)\right)^{-1}\cdot\right.$$
$$\left.\left(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2]\right)\right)\text{-}0.5\ln|\rho(h)| \qquad (4.9)$$

$$=\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(\left\{\frac{1}{\sigma_j}\left[1+\xi_j\left(\frac{x_{ji}-\mu_j}{\sigma_j}\right)\right]^{-\frac{1}{\xi_j}}\right\}\exp\left\{-\left[1+\xi_j\left(\frac{x_{ji}-\mu_j}{\sigma_j}\right)\right]^{-\frac{1}{\xi_j}}\right\}\cdot\right.$$
$$\left.\left\{\frac{1}{\sigma_k}\left[1+\xi_k\left(\frac{x_{ki}-\mu_k}{\sigma_k}\right)\right]^{-\frac{1}{\xi_k}}\right\}\exp\left\{-\left[1+\xi_k\left(\frac{x_{ki}-\mu_k}{\sigma_k}\right)\right]^{-\frac{1}{\xi_k}}\right\}\right).$$
$$\left(\frac{1}{2}\left(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2]\right)^T.\left(\rho(h)\right)^{-1}\cdot\left(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2]\right)\right)\text{-}0.5\ln|\rho(h)|$$

didalam copula terdapat variabel random u yang merupakan hasil transformasi x, dengan fungsi transformasi mengikuti persamaan (4.10) sebagai berikut:

$$u_j = F_j\left(x_{ij}\right) = \exp\left\{-\left[1+\xi_j\left(\frac{x_{ij}-\mu_j}{\sigma_j}\right)\right]^{-\frac{1}{\xi_j}}\right\} \qquad (4.10)$$

sehingga bentuk *ln pairwise* dapat ditulis kembali dengan menjabarkan variabel u, sehingga persamaan (4.9) menjadi seperti persamaan (4.11) sebagai berikut:

$$\ell_p\left(\hat{\boldsymbol{\beta}}\right)=\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(f_{x_j}\left(x_{ji}\right)\cdot f_{x_k}\left(x_{ki}\right)\right)\cdot\left(\frac{1}{2}\left(\Phi^{-1}\left(u_{ji}\right)\ \Phi^{-1}\left(u_{ki}\right)\right)^T\cdot\left(\rho(h)^{-1}\right)\cdot\right.$$
$$\left.\left(\Phi^{-1}\left(u_{ji}\right)\ \Phi^{-1}\left(u_{ki}\right)\right)\right)\text{-}0.5\ln|\rho(h)| \qquad (4.11)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \ln \left(\left\{ \frac{1}{\sigma_j} \left[1 + \xi_j \left(\frac{x_{ji} - \mu_j}{\sigma_j} \right) \right]^{-\frac{1}{\xi_j}} \right\} \exp \left\{ - \left[1 + \xi_j \left(\frac{x_{ji} - \mu_j}{\sigma_j} \right) \right]^{-\frac{1}{\xi_j}} \right\} . \right.$$

$$\left. \left\{ \frac{1}{\sigma_k} \left[1 + \xi_k \left(\frac{x_{ki} - \mu_k}{\sigma_k} \right) \right]^{-\frac{1}{\xi_k}} \right\} \exp \left\{ - \left[1 + \xi_k \left(\frac{x_{ki} - \mu_k}{\sigma_k} \right) \right]^{-\frac{1}{\xi_k}} \right\} \right) .$$

$$\left[\begin{array}{l} \frac{1}{2} \left(\Phi^{-1} \left(\exp \left\{ - \left[1 + \xi_j \left(\frac{x_{ji} - \mu_j}{\sigma_j} \right) \right]^{-\frac{1}{\xi_j}} \right\} \right) \Phi^{-1} \left(\exp \left\{ - \left[1 + \xi_k \left(\frac{x_{ki} - \mu_k}{\sigma_k} \right) \right]^{-\frac{1}{\xi_k}} \right\} \right) \right)^T \\ \cdot \left(\rho(h)^{-1} \right) \cdot \\ \left(\Phi^{-1} \left(\exp \left\{ - \left[1 + \xi_j \left(\frac{x_{ji} - \mu_j}{\sigma_j} \right) \right]^{-\frac{1}{\xi_j}} \right\} \right) \Phi^{-1} \left(\exp \left\{ - \left[1 + \xi_k \left(\frac{x_{ki} - \mu_k}{\sigma_k} \right) \right]^{-\frac{1}{\xi_k}} \right\} \right) \right) \end{array} \right]$$

$$-0.5 \ln |\rho(h)|$$

Dalam *spatial extreme value* dibentuk model persamaan *trend surface*, dengan bentuk dari model *trend surface* mengikuti persamaan (4.12) sebagai berikut

$$\begin{aligned} \hat{\mu}(j) &= \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1} u(j) + \hat{\beta}_{\mu,2} v(j) \\ \hat{\sigma}(j) &= \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1} u(j) + \hat{\beta}_{\sigma,2} v(j) \\ \hat{\xi}(j) &= \beta_{\xi,0} \end{aligned} \tag{4.12}$$

Untuk melakukan proses estimasi GEV secara *spatial* ketiga parameter $\hat{\mu}(j)$, $\hat{\sigma}(j)$, dan $\hat{\xi}(j)$ tersebut dapat ditulis kembali menjadi bentuk matriks mengikuti persamaan (4.13) sebagai berikut:

$$\begin{aligned} \mu(j) &= \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\mu} \\ \sigma(j) &= \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\sigma \\ \xi(j) &= \beta_\xi = \beta_{0,\xi} \end{aligned} \tag{4.13}$$

dengan:

$$\mathbf{d_j} = \begin{bmatrix} 1 \\ u(j) \\ v(j) \end{bmatrix} \qquad \boldsymbol{\beta_\mu} = \begin{bmatrix} \beta_{\mu,0} \\ \beta_{\mu,1} \\ \beta_{\mu,2} \end{bmatrix} \qquad \boldsymbol{\beta_\sigma} = \begin{bmatrix} \beta_{\sigma,0} \\ \beta_{\sigma,1} \\ \beta_{\sigma,2} \end{bmatrix}$$

dimana

$u(j)$: *longtitude* dari suatu lokasi *j*

$v(j)$: *latitude* dari suatu lokasi *j*

Berdasarkan bentuk parameter μ, σ dan ξ tersebut, maka bentuk fungsi *ln pairwise likelihood* dapat dijabarkan kembali seperti persamaan (4.14) berikut:

$$\ell_p\left(\hat{\boldsymbol{\beta}}\right) = \sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \ln \Delta$$

$$\Delta = \left(\left\{ \frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\sigma}} \left[1 + \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi}} \right\} \exp\left\{ - \left[1 + \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi}} \right\} \cdot \right.$$

$$\left. \left\{ \frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\sigma}} \left[1 + \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{d_k^T \beta_\xi}} \right\} \exp\left\{ - \left[1 + \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi}} \right\} \right) \cdot$$

$$\frac{1}{2} \cdot \left(\Phi^{-1} \left(\exp\left\{ - \left[1 + \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi}} \right\} \right) \right.$$

$$\left. \Phi^{-1} \left(\exp\left\{ - \left[1 + \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi}} \right\} \right) \right)^T \cdot \left(\rho(h)^{-1} \right) \cdot (**)$$

$$(**) = \Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)$$

$$\left.\left.\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\right) \text{-}0.5\ln|\rho(h)| \tag{4.14}$$

Selanjutnya untuk melakukan penurunan fungsi *ln pairwise likelihood* terhadap parameter $\boldsymbol{\beta}_{\boldsymbol{\mu}}, \boldsymbol{\beta}_{\boldsymbol{\sigma}}$ dan β_{ξ}, fungsi *ln pairwise likelihood* dimisalkan sebagai berikut:

$$l\left(\widehat{\boldsymbol{\beta}}\right) = (a \cdot b - c)$$

dimana

$$a = \ln\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.$$

$$\left.\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)$$

$$b = \frac{1}{2}\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{T}$$

$$\cdot\left(\rho(h)^{-1}\right)\cdot$$

$$\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)$$

$$c = \text{-}0.5\ln|\rho(h)|$$

Untuk penurunan fungsi *ln pairwise likelihood* terhadap parameter $\boldsymbol{\beta}_{\boldsymbol{\mu}}$ adalah menurukan $a,\ b,\ \text{dan}\ c$ terhadap $\boldsymbol{\beta}_{\boldsymbol{\mu}}$, kemudian menyamadenganakan dengan vektor nol mengikuti persamaan berikut:

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \frac{\partial (a \cdot b - c)}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = 0$$

$$= \frac{\partial a}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} b + a \frac{\partial b}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} - \frac{\partial c}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = 0$$

dimana

$$\begin{aligned}
\frac{\partial a}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = & \left[\left(\frac{1}{\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}} \left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \right) \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \right\} \cdot \right. \right. \\
& \left. \left. \cdot \left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(-\mathbf{d}_{\mathbf{j}} \right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right) + \left(\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \right\} . \left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(\text{-} \mathbf{d}_{\mathbf{j}} \right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right) \right] + \\
& \left[\left(\frac{1}{\left\{ \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}} \left\{ \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \right) \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \right\} . \right. \right. \\
& \left. \left. \left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(-\mathbf{d}_{\mathbf{k}} \right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right) + \left(\left\{ \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \right\} . \left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(-\mathbf{d}_{\mathbf{k}} \right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right) \right]
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial b}{\partial \boldsymbol{\beta}_\mu} = \frac{1}{2}\Bigg[\Bigg[& \phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg).\exp\Bigg\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\Bigg\}. \\
& \frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}-1}.\left(\frac{\mathbf{d}_j^T\boldsymbol{\beta}_\xi.(-\mathbf{d}_j)}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\Bigg] \\
& \Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{d_k^T\beta_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg).\exp\Bigg\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\Bigg\}. \\
& \frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}-1}.\left(\frac{\mathbf{d}_k^T\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\Bigg]^T\cdot\left(\rho(h)^{-1}\right). \\
& \Bigg(\Phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg)\Phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg)\Bigg)+ \\
& \Bigg(\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg).\exp\Bigg\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\Bigg\} \\
& .\left(\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{d_j^T\beta_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}-1}\right).\left(\frac{\mathbf{d}_j^T\boldsymbol{\beta}_\xi.(-\mathbf{d}_j)}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\ \phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg) \\
& .\exp\Bigg\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\Bigg\}.\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}-1}.\left(\frac{\mathbf{d}_k^T\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\Bigg) \\
& \Bigg(\Phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg)\Phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg)\Bigg)^T \\
& \cdot\left(\rho(h)^{-1}\right)\cdot
\end{aligned}
$$

$$
\frac{\partial c}{\partial \boldsymbol{\beta}_\mu} = 0
$$

Untuk hasil penurunan $\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_\mu}$ secara detail dapat dilihat pada Lampiran 1. Untuk penurunan fungsi *ln pairwise likelihood* terhadap parameter $\boldsymbol{\beta}_\sigma$, memisalkan

a, b, dan c seperti penurunan sebelumnya. Kemudian menurukan a, b, dan c terhadap $\boldsymbol{\beta}_{\sigma}$, dan menyamadengankan dengan vektor nol sehingga diperoleh

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_{\sigma}} = \frac{\partial (a \cdot b - c)}{\partial \boldsymbol{\beta}_{\sigma}} = 0$$

$$= \frac{\partial a}{\partial \boldsymbol{\beta}_{\sigma}} b + a \frac{\partial b}{\partial \boldsymbol{\beta}_{\sigma}} - \frac{\partial c}{\partial \boldsymbol{\beta}_{\sigma}} = 0$$

dimana

$$\frac{\partial a}{\partial \boldsymbol{\beta}_{\sigma}} = \left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}-1}\right\}.\right.\right.$$

$$\left.\left(\frac{\mathbf{d}_{j}}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)\right)+$$

$$\left.\left(\left\{\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}-1}\right\}.\left(-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{j}\left(x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)\right)\right]+$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}}-1}\right\}.\right.\right.$$

$$\left.\left(\frac{\mathbf{d}_{j}}{\left(\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)\right)+$$

$$\left.\left(\left\{\frac{1}{\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}}-1}\right\}.\left(-\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{k}\left(x_{ki}-\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{k}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)\right)\right]$$

$$\frac{\partial b}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \frac{1}{2}\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.\right.$$

$$\left.\left.\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right]\right]$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\ \exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.$$

$$\left.\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right]^{T}$$

$$\cdot\left(\rho\left(h\right)^{-1}\right).$$

$$\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)+$$

$$\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.$$

$$\left.\left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{j}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right.$$

$$
\cdot \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}} - 1} \cdot \left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{k}} \left(x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right) \Bigg]
$$

$$
\cdot \left(\Phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \right) \Phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{T}
$$

$$
\cdot \left(\rho(h)^{-1} \right)
$$

$$
\frac{\partial c}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = 0
$$

untuk Hasil penurunan $\frac{\partial l(\hat{\boldsymbol{\beta}})}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}}$ secara detail dapat dilihat pada Lampiran 2. Untuk

Penurunan *ln pairwise likelihood* terhadap parameter β_{ξ} sama seperti cara permisalan dan penurunannya penurunan $\boldsymbol{\beta}_{\boldsymbol{\mu}}$ dan $\boldsymbol{\beta}_{\boldsymbol{\sigma}}$ Dimana $\frac{\partial a}{\partial \beta_{\xi}}$, $\frac{\partial b}{\partial \beta_{\xi}}$, dan $\frac{\partial c}{\partial \beta_{\xi}}$ beserta persamaan secara detail $\frac{\partial l(\hat{\boldsymbol{\beta}})}{\partial \beta_{\xi}}$ ada pada Lampiran 3.

Dari hasil estimasi parameter tersebut memberikan hasil yang tidak *close form,* sehingga estimasi parameter harus dilanjutkan menggunakan iterasi numerik. Iterasi Numerik yang digunakan dalam penelitian ini adalah Nelder-Mead. Penelitian ini menggunakan iterasi Nelder-Mead karena pada penelitian sebelumnya nilai akurasi Nelder-Mead lebih baik daripada BFGS (*Broyden-Fletcher Goldfarb-Shanno*) Quasi Newton, sementara BFGS sendiri merupakan metose iterasi perbaikan Newton Raphson sehingga hasil BFGS lebih baik daripada Newton Raphson. Karena meggunakan metode iterasi Nelder-Mead maka Fungsi *ln pairwise likelihood* dapat dituliskan $\ln\left(L_p\left(\boldsymbol{\beta}_{\boldsymbol{\mu}}, \boldsymbol{\beta}_{\boldsymbol{\sigma}}, \beta_{\xi}\right)\right) = \ell_p\{\boldsymbol{\psi}\}$ dimana $\boldsymbol{\psi} = \left(\boldsymbol{\beta}_{\boldsymbol{\mu}}, \boldsymbol{\beta}_{\boldsymbol{\sigma}}, \beta_{\xi}\right)$. Untuk memaksimumkan fungsi *ln pairwise likelihood* $\ell_p\{\psi\}$ dimana $\ell_p \in R^3$, maka *initial point* yang digunakan yaitu ada sebanyak 3+1= 4 yaitu $\psi_1, \ldots, \psi_4$. Langkah-langkahnya sebagai berikut:

1. Substitusi nilai $\psi_1, \dots, \psi_4$ ke dalam fungsi $\ell_p(\psi)$, kemudian diurutkan mulai nilai terbesar sampai terkecil $\ell_p(\psi_1) \geq \ell_p(\psi_2) \geq \cdots \geq \ell_p(\psi_4)$ sehingga ψ_1 disebut titik terbaik (*best*) dan ψ_4 disebut titik terburuk (*worst*).
2. Menentukan nilai ψ_o, yaitu nilai *centroid* (tengah) pada setiap *initial point* kecuali ψ_4.
3. Tahap *Reflection*
   - Menentukan titik refleksi ψ_r dengan rumus $\psi_r = \psi_o + a(\psi_o - \psi_4)$, kemudian substitusi nilai ψ_r ke dalam fungsi $\ell_p(\psi)$ sehingga ada tiga kemungkinan kondisi yang dicapai oleh $\ell_p(\psi_r)$.
   - Kondisi-1: jika ψ_r memenuhi kondisi $\ell_p(\psi_1) \geq \ell_p(\psi_r) > \ell_p(\psi_m)$, maka $\psi_4 = \psi_r$ dan kembali ke langkah-1.
4. Tahap *Expansion*
   - Kondisi-2: jika ψ_r memenuhi kondisi $\ell_p(\psi_r) > \ell_p(\psi_1)$, maka menentukan titik ekspansi ψ_e dengan rumus $\psi_e = \psi_o + b(\psi_o - \psi_4)$, kemudian substitusi nilai ψ_e ke dalam fungsi $\ell_p(\psi)$.
   - Selanjutnya jika titik ψ_e memenuhi kondisi $\ell_p(\psi_e) > \ell_p(\psi_1)$, maka ψ_4 = ψ_e dan kembali ke langkah-1. Sedangkan jika titik ψ_e tidak memenuhi kondisi tersebut, maka $\psi_4 = \psi_r$, dan kembali ke langkah-1.
5. Tahap *Contraction*
   - Kondisi-3 : jika ψ_r memenuhi kondisi $\ell_p(\psi_r) \leq \ell_p(\psi_3)$, maka menentukan titik kontraksi ψ_c dengan rumus $\psi_c = \psi_o + c(\psi_o - \psi_4)$, kemudian substitusi nilai ψ_c ke dalam fungsi $\ell_p(\psi)$
   - Jika titik ψ_c memenuhi kondisi $\ell_p(\psi_c) > \ell_p(\psi_4)$, maka $\psi_4 = \psi_c$ dan kembali ke langkah-1.
6. Tahap *Reduction*

   Pada tahap ini jika ψ_r tidak memenuhi salah satu dari tiga kondisi tersebut, maka untuk setiap titik (kecuali titik terbaik ψ_1) diganti menggunakan rumus :

$\psi_i = \psi_1 + d(\psi_i - \psi_1)$ dimana $i \in \{2,3,4\}$

Catatan: *a*, *b*, *c*, dan *d* adalah koefisien *reflection*, *expansion*, *contraction*, dan *shrink* dengan domain $a > 0$, $b > 1$, $0 < c < 1$, dan $0 < d < 1$. Nilai standar digunakan untuk koefisien-koefisien tersebut yaitu $a = 1$, $b = 2$, $c = -½$, dan $d = ½$. Secara umum metode ini membuat sebuah segi banyak dalam ruang variabelnya yang terus diiterasi sehingga segi banyak itu semakin lama makin mengecil. Pada saat segi banyak tersebut menjadi sangat kecil sekali, maka didapatkan hasil yang optimum, yang ditentukan nilainya sebagai suatu nilai konvergensi. (Nelder dan Mead, 1965).

4.2 Penyusunan Model Curah Hujan Ekstrem di Kabupaten Ngawi

Dari estimasi parameter pada Sub Bab 4.1 diaplikasikan pada data curah hujan harian Kabupaten Ngawi dengan satuan data adalah mm/hari. Pertimbangan penerapan terhadap data telah dijelaskan pada sub bab latar belakang. Kabupaten Ngawi memiliki 24 Pos hujan yang tersebar di seluruh wilayah Kabupaten. Penelitian ini mengeliminasi 7 Pos hujan yang tidak termasuk dalam satu ZOM. Perbedaan ZOM menyebabkan kecenderungan pola hujan berbeda/heterogen, yang dapat menyebabkan analisis *spatial* tidak tepat. Pos-pos hujan yang tidak termasuk dalam satu Zona Musim ini adalah pos Tretes, Begal, Bekoh, Babadan, Jogorgo, Ngrambe, Kedung Urung-urung. Enam Pos hujan lainnya dieliminasi dengan pertimbangan terlalu banyak data yang irasional pada pos tersebut. Data irasional yang dimaksud seperti data bernilai nol pada lebih dari satu tahun, yang mengakibatkan data tersebut tidak dapat didekati dengan nilai pada tahun tahun sebelumnya. Berdasarkan pertimbangan tersebut, penerapan estimasi pada data curah hujan Kabupaten Ngawi ini hanya melibatkan sebelas pos hujan, yaitu Pos Gemarang, Guyung, Karangjati, Kedunggalar, Kedungbendo, Kricak, Kendal, Mardisari, Mantingan, Papungan, Paron. Sekilas data curah hujan sebelas Pos hujan dapat dilihat pada Lampiran 4.

4.2.1 Deskripsi Curah Hujan di Kabupaten Ngawi

Deskripsi curah hujan di sebelas Pos hujan perlu dilakukan sebagai informasi awal untuk mengetahui karakteristik atau gambaran umum dan pola curah hujan yang digunakan. Deskripsi dari data curah hujan Kabupaten Ngawi dari bulan Desember tahun 1990 sampai dengan bulan November tahun 2015 disajikan pada Tabel 4.1 sebagai berikut:

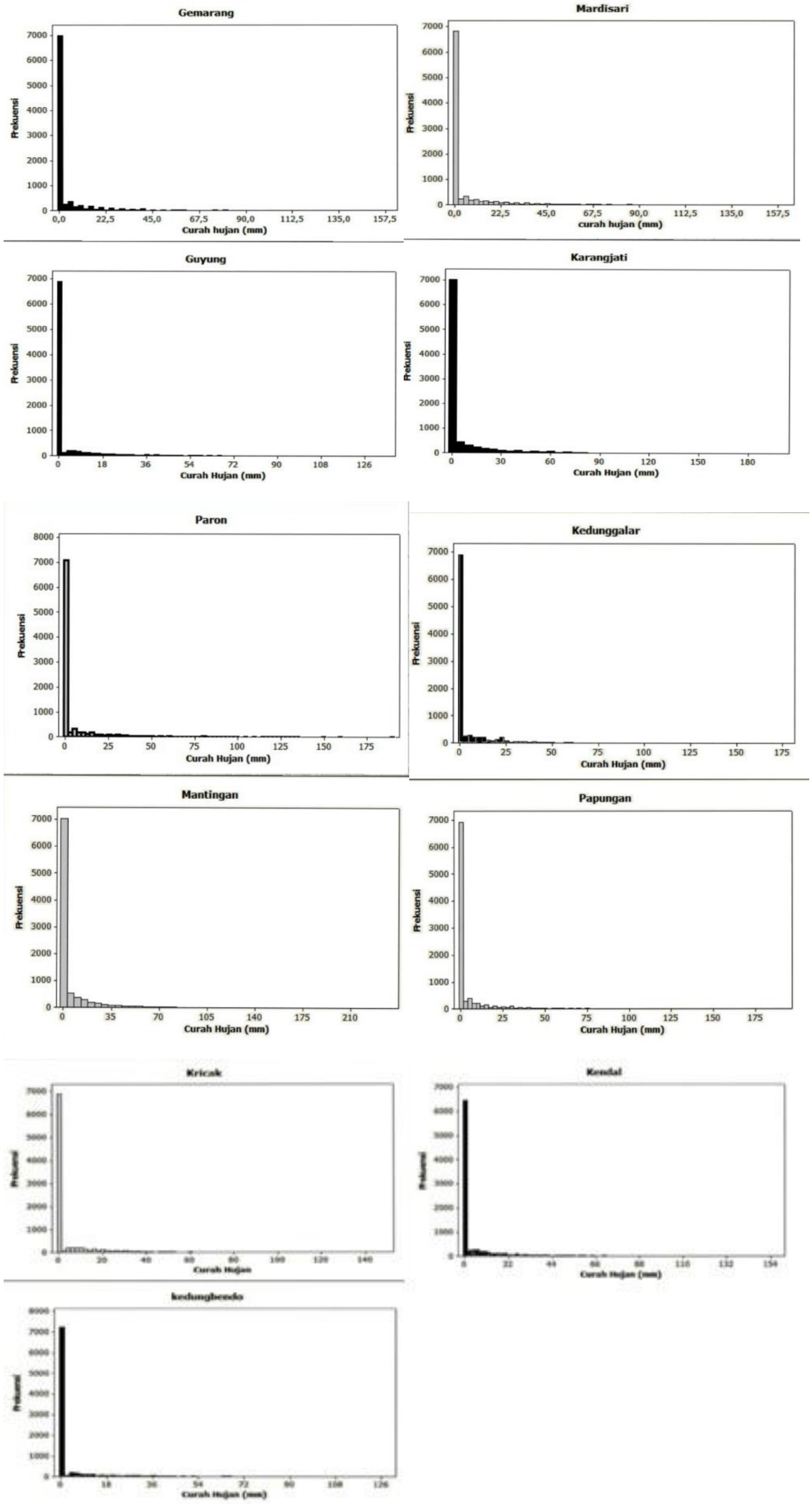
Tabel 4.1 Deskripsi Data Curah Hujan 11 Pos Hujan Kabupaten Ngawi (mm/hari)

No	Pos Hujan	Rata-rata	Min	Max	*Skewnes*	Kurtosis
1	Gemarang	4,868	0	160	4,39	23,69
2	Guyung	5,908	0	136	3,88	17,41
3	Karangjati	6,250	0	201	3,52	15,30
4	Kedungbendo	4,626	0	129	4,18	20,50
5	Kedunggalar	4,629	0	178	4,26	25,72
6	Kendal	5,949	0	158	3,94	19,88
7	Kricak	4,946	0	150	3,98	20,68
8	Mantingan	5,358	0	240	4,37	27,72
9	Mardisari	5,663	0	162	4,07	20,18
10	Papungan	4,964	0	193	4,21	23,23
11	Paron	5,127	0	190	4,32	24,22

Tabel 4.1 menunjukkan bahwa di Kabupaten Ngawi Curah hujan minimum adalah nol di seluruh Pos hujan, yang artinya tidak ada curah hujan sama sekali dalam satu hari. Pos hujan yang memiliki intensitas curah hujan terendah adalah Pos hujan Kedungbendo dengan rata-rata curah hujan 4,626 mm/hari dan Pos hujan yang memiliki intensitas curah hujan tertinggi adalah pos hujan Karangjati dengan intensitas rata-rata curah hujan adalah 6,250 mm. Curah hujan maksimum sebesar 240/hari mm dalam satu hari telah terjadi pada Pos hujan Mantingan, yang berarti hujan dengan curah terekstrem telah terjadi pada wilayah ini. Nilai maksimum yang terkecil yaitu 129 terjadi pada Pos hujan Kedungbendo mengindikasikan bahwa pada sebelas pos hujan di atas telah terjadi hujan yang tergolong ekstrem berdasarkan dengan definisi dari BMKG, yang menyatakan bahwa curah hujan dikategorikan ekstrem apabila mencapai 100 mm/hari. Alasan diperkuat dengan nilai *skewness* yang diperoleh pada kesebelas Pos Hujan cukup besar. Nilai *skewness* ini menyatakan distribusi data cenderung simetri/miring ke salah satu sisi (sisi kanan atau kiri). Dikaitkan dengan analisis secara visual pada

histogram data curah hujan masing-masing pos pada gambar 4.1. Tabel 4.1 juga menyajikan nilai kurtosis, nilai kurtosis memberikan gambaran seberapa runcing kurva distribusi data. Semakin besar nilai kurtosisnya, semakin runcing kuva yang mengindikasikan bahwa keragaman data cenderung lebih kecil.

Adanya data ekstrem dan pola data *heavytail* pada sebelas Pos hujan juga dapat ditunjukkan dengan analisis secara visual pada histogram data curah hujan masing-masing Pos yang terlihat pada Gambar 4.1. Gambar 4.1 menunjukkan bahwa kurva distribusi data miring ke kanan dan memperlihatkan tingginya frekuensi data menonjol di sekitar nilai nol, sedangkan masih terdapat kejadian dengan curah hujan yang jauh lebih besar dari nol dengan frekuensi yang jauh lebih kecil, sehingga mengindikasikan adanya pola data *heavy tail*. Berdasarkan alasan tersebut sebelas Pos hujan ini dikategorikan layak menjadi objek penelitian karena merupakan data *heavy tail*, sehingga dapat dilakukan pengambilan sampel ekstremnya. Adanya indikasi nilai ekstrem dan data *heavy tail* ini menunjukkan bahwa data curah hujan harian tidak berdistribusi normal dan menyebabkan pada penelitian ini menggunakan metode *extreme value theory*.



Gambar 4.1 Histogram Data Curah Hujan Harian 11 Pos Hujan

4.2.2 Penentuan Data Sampel dengan *Block Maxima*

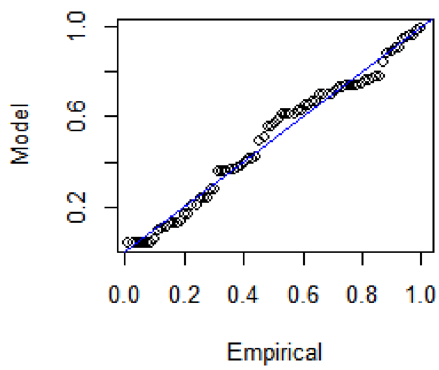
Penentuan ukuran blok dalam metode *block maxima* merupakan hal yang sulit seperti halnya menentukan nilai *threshold* dalam metode *Peaks Over Threshold* (POT). Permasalahan tersebut dapat menghasilkan taksiran parameter yang bias dan nilai varians yang besar jika ukuran blok terlalu kecil atau terlalu panjang (Coles, 2001). Data sampel pada penelitian ini merupakan nilai-nilai ekstrem dari data curah hujan pada sebelas Pos hujan di Kabupaten Ngawi. Penentuan sampel dilakukan menggunakan metode *block maxima* berdasarkan acuan BMKG yang mengklasifikasikan pola hujan monsun pada sebagian besar wilayah di Pulau Jawa. Metode *block maxima* dilakukan dengan membagi data ke dalam blok periode tiga bulanan, *block* yang terbentuk yaitu Desember-Januari-Februari (DJF), Maret-April-Mei (MAM), Juni-Juli-Agustus (JJA), dan September-Oktober-November (SON). Pembagian data dalam waktu tiga bulanan ini didasarkan pada pola curah hujan di sebelas Pos hujan di Kabupaten Ngawi yang berpola monsun. Pada pola monsun, pembagian periode musimnya meliputi DJF yang merupakan periode musim hujan, MAM merupakan periode transisi dari musim hujan ke musim kemarau, JJA merupakan periode musim kemarau, dan SON merupakan periode transisi dari musim kemarau ke musim hujan. Selama periode sampel (1990-2015) terbentuk 100 blok. Dari satu blok diambil satu nilai ekstrem, nilai ekstrem yang diambil merupakan nilai maksimum dari masing-masing blok. Berdasarkan langkah-langkah tersebut terambil 100 data yang merupakan nilai maksimum dari setiap blok tiga bulanan, dari sebanyak 9131 data curah hujan masing-masing pos. Data sampel ekstrem untuk data *training* terlampir pada Lampiran 5. Data sampel *training* terdiri dari 79 blok dengan periode dimulai bulan Desember tahun 1990 sampai Agustus tahun 2015. Sedangkan data sampel ekstrem untuk data *testing* terlampir pada Lampiran 6. Data sampel yang *testing* terdiri dari 21 blok dengan periode dimulai bulan September tahun 2010 sampai bulan Nopember tahun 2015. Selanjutnya menunjukkan deskriptif untuk data curah hujan yang diperoleh dengan *block maxima* 3 bulan pada Tabel 4.

Tabel 4.2 Deskriptif Data Curah Hujan Ekstrem 11 Pos Hujan Kabupaten Ngawi (mm/hari)

No	Pos Hujan	Rata-rata	Min	Max	Median
1	Gemarang	68,69	0	160	76
2	Guyung	69,69	0	136	77
3	Karangjati	72,07	0	201	70
4	Kedungbendo	52,93	0	129	49
5	Kedunggalar	59,79	0	178	59
6	Kendal	74,48	0	158	82
7	Kricak	62,88	0	150	68
8	Mantingan	71,55	0	240	70
9	Mardisari	72,93	0	162	75
10	Papungan	68,12	0	193	74
11	Paron	67,46	0	161	67

4.2.3 Uji Kesesuaian Distribusi

Data ekstrem yang diperoleh harus terlebih dahulu di uji apakah data ekstrem yang di ambil dari *block maxima* berdistribusi GEV. *Probability plot* digunakan untuk menunjukkan bahwa sampel ekstrem periode blok tiga bulanan berdistribusi GEV dan melalui pengujian kesesuaian distribusi GEV dengan uji *Anderson Darling*. Gambar 4.2 ditampilkan *probability plot* dari pos hujan Gemarang dengan blok tiga bulan



Gambar 4.2 *Probability plot* pos hujan Gemarang

Gambar 4.2 menunjukkan bahwa hampir semua titik sebaran mengikuti garis linier. Hal ini menunjukkan bahwa sampel ekstrem di Pos hujan Gemarang telah mengikuti distribusi GEV. Pola yang sama terlihat pada Lampiran 18 dimana hampir semua titik sebaran mengikuti garis linier terjadi di sepuluh Pos hujan lainnya. Hal ini berarti bahwa sampel ekstrem di sebelas Pos hujan Kabupaten Ngawi telah mengikuti distribusi GEV.

Selanjutnya untuk mendukung kesimpulan tersebut, dilakukan pengujian *Anderson Darling* dengan pengujian hipotesis sebagai berikut:

H0 : $F(x) = F^*(x)$ (Data mengikuti distribusi teoritis $F^*(x)$).

H1: $F(x) \neq F^*(x)$ (Data tidak mengikuti distribusi teoritis $F^*(x)$)

Statistik uji yang digunakan yaitu pada persamaan (2.12) dengan menggunakan tingkat signifikansi $\alpha = 5\%$, tolak H0 jika A_n^2 lebih besar dari A_{tabel}^2. A_{tabel}^2 dapat dilihat pada Lampiran 7 atau menggunakan kriteria p-value < α .

Tabel 4.3 Uji Anderson Darling 11 Pos Hujan

No	Pos Hujan	A_n^2	p-value	Keputusan
1	Gemarang	0,663	0,948	Gagal Tolak H_0
2	Guyung	0,567	0,864	Gagal Tolak H_0
3	Karangjati	0,955	1,000	Gagal Tolak H_0
4	Kedungbend o	0,495	0,863	Gagal Tolak H_0
5	Kedunggalar	0,473	0,856	Gagal Tolak H_0
6	Kendal	0,745	0,967	Gagal Tolak H_0
7	Kricak	0,718	0,964	Gagal Tolak H_0
8	Mantingan	0,921	0,996	Gagal Tolak H_0
9	Mardisari	0,257	0,345	Gagal Tolak H_0
10	Papungan	0,951	0,995	Gagal Tolak H_0
11	Paron	0,313	0,523	Gagal Tolak H_0

Tabel 4.3 menunjukkan hasil pengujian kesesuaian distribusi sampel ekstrem dengan metode *block maxima* periode blok tiga bulanan sudah mengikuti distribusi GEV. Hal tersebut dapat dilihat dari nilai p-value > α sehingga menghasilkan keputusan gagal tolak H0.

4.2.4 Dugaan Nilai Parameter GEV Univariat

Sampel ekstrem yang diperoleh selanjutnya digunakan untuk mengestimasi parameter distribusi GEV. Data ekstrem yang diperoleh dari *blok maxima* kemudian digunakan untuk menaksir parameter GEV univariat yaitu $\hat{\mu}$ sebagai parameter lokasi, $\hat{\sigma}$ sebagai parameter skala, dan $\hat{\xi}$ sebagai parameter bentuk. Dimana parameter yang akan ditaksir merupakan parameter perlokasi berdasarkan periode blok tiga bulan. Parameter $\hat{\mu}$, $\hat{\sigma}$, dan $\hat{\xi}$ dihasilkan dari proses estimasi menggunakan MLE, karena parameter yang dihasilkan tidak *close form* maka digunakan metode numerik yaitu Nelder-Mead. Hasil estimasi parameter GEV disajikan pada Tabel 4.4 sebagai berikut

Tabel 4.4 Nilai Parameter $\hat{\mu}$, $\hat{\sigma}$, dan $\hat{\xi}$ GEV *univariat*

No	Pos Hujan	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\xi}$
1	Gemarang	55,281	39,331	-0,305
2	Guyung	60,876	41,886	-0,534
3	Karangjati	55,731	39,663	-0,190
4	Kedungbendo	39,402	32,248	-0,200
5	Kedunggalar	44,910	30,349	-0,097
6	Kendal	62,065	39,861	-0,349
7	Kricak	50,411	36,597	-0,304
8	Mantingan	53,988	35,691	-0,080
9	Mardisari	59,540	37,578	-0,278
10	Papungan	57,900	39,524	-0,214
11	Paron	53,568	38,550	-0,276

Tabel 4.4 menunjukkan distribusi data curah hujan ekstrem di sebelas pos hujan pengamatan di Kabupaten Ngawi berdistribusi *reversed* weibull dikarenakan parameter bentuk bernilai negatif $(\xi < 0)$. Untuk melakukan perhitungan *spatial* dengan pendekatan copula data perlu dilakukan transformasi ke distribusi frechet karena distribusi frechet memiliki ekor yang paling *heavytail* dibandingkan distribusi GEV yang lain dan copula lebih tepat diterapkan untuk kasus *heavytail*. Proses transformasi data ke distribusi frechet dinamakan proses *max-stable*. Setelah data ditransformasi ke frechet data perlu ditransformasi lagi ke copula seperti pada penjelasan pada Sub bab 4.1.

4.2.5 Dependensi *Spatial* Curah Hujan Ekstrem

Pada kasus *spatial* ekstrem, salah satu analisis yang menarik adalah mengetahui ukuran dependensi *spatial* pada lokasi, yaitu dengan koefisien ekstremal. Koefisien ekstermal merupakan ukuran dependensi ekstremal multivariate yang dikemukakan oleh Smith (1990). Koefisien ekstremal menggambarkan dependensi *spatial* ekstrem secara parsial atau bivariat (dua lokasi berpasangan). Perhitungan koefisien ekstermal menggunakan persamaan (2.14). Dalam perhitungan koefisien ekstermal, diperlukan informasi mengenai jarak antar Pos hujan yang dihitung menggunakan konsep jarak Euclid. Dalam penelitian ini sebanyak sebelas Pos hujan yang diteliti, dan berdasarkan hasil perhitungan jarak antar dua Pos hujan, tidak ada pasangan Pos hujan yang mempunyai jarak yang sama, sehingga terdapat 55 pasang lokasi yang perlu diestimasi nilai koefisien ekstremalnya. Estimasi koefisien ekstermal sebanyak 55 pasang terdapat pada Tabel 4.5 sebagai berikut:

Tabel 4.5. Koefisien Ekstermal antar lokasi pengamatan

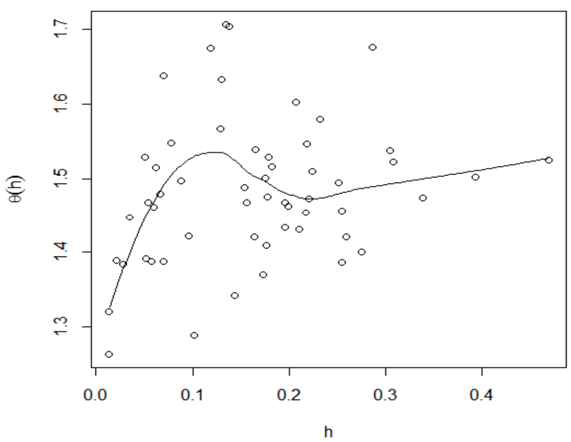
No	Pasangan 2 lokasi	Jarak Euclid	Koefisien Ekstermal
1	Gemarang-Guyung	0,118	1,675
2	Gemarang-Karangjati	0,255	1,456
3	Gemarang-Kedungbendo	0,177	1,476
4	Gemarang- Kedunggalar	0,054	1,467
5	Gemarang-Kendal	0,181	1,516
6	Gemarang-Kricak	0,021	1,388
7	Gemarang-Mantingan	0,216	1,453
8	Gemarang-Mardisari	0,051	1,391
9	Gemarang-Papungan	0,014	1,321
10	Gemarang-Paron	0,051	1,529
11	Guyung - Karangjati	0,207	1,602
12	Guyung - Kedungbendo	0,178	1,529
13	Guyung - Kedunggalar	0,138	1,704
14	Guyung-Kendal	0,134	1,706
15	Guyung -Kricak	0,129	1,633
16	Guyung -Mantingan	0,287	1,676
17	Guyung -Mardisari	0,077	1,548
18	Guyung -Papungan	0,129	1,567
19	Guyung-Paron	0,070	1,637
20	Karangjati - Kedungbendo	0,102	1,288
21	Karangjati - Kedunggalar	0,304	1,537
22	Karangjati-Kendal	0,339	1,474
23	Karangjati -Kricak	0,276	1,401
24	Karangjati -Mantingan	0,469	1,525
25	Karangjati -Mardisari	0,209	1,431
26	Karangjati -Papungan	0,255	1,386
27	Karangjati Paron	0,218	1,546
28	Kedungbendo-Kedunggalar	0,231	1,580
29	Kedungbendo -Kendal	0,308	1,522
30	Kedungbendo -Kricak	0,198	1,463
31	Kedungbendo -Mantingan	0,393	1,502
32	Kedungbendo -Mardisari	0,143	1,342
33	Kedungbendo -Papungan	0,173	1,369
34	Kedungbendo -Paron	0,156	1,468

35	Kedunggalar - Kendal	0,154	1,487

Tabel 4.5 (Lanjutan)

No	Pasangan 2 lokasi	Jarak Euclid	Koefisien Ekstermal
36	Kedunggalar -Kricak	0,035	1,447
37	Kedunggalar -Mantingan	0,165	1,539
38	Kedunggalar -Mardisari	0,095	1,421
39	Kedunggalar -Papungan	0,062	1,514
40	Kedunggalar -Paron	0,088	1,497
41	Kendal - Kricak	0,175	1,501
42	Kendal -Mantingan	0,223	1,509
43	Kendal -Mardisari	0,176	1,409
44	Kendal -Papungan	0,195	1,468
45	Kendal -Paron	0,163	1,420
46	Kricak - Mantingan	0,195	1,434
47	Kricak -Mardisari	0,070	1,388
48	Kricak -Papungan	0,027	1,384
49	Kricak -Paron	0,067	1,479
50	Mantingan - Mardisari	0,260	1,421
51	Mantingan -Papungan	0,220	1,473
52	Mantingan -Paron	0,251	1,495
53	Mardisari -Papungan	0,058	1,388
54	Mardisari -Paron	0,013	1,263
55	Papungan -Paron	0,060	1,461

Nilai koefisien ekstremal yang diperoleh kemudian diplotkan terhadap jarak *h* sehingga menghasilkan plot seperti pada Gambar 4.3.



Gambar 4.3 Koefisien Ekstermal

Gambar 4.3 menunjukan bahwa titik-titik menyebar di sekitar nilai 1,2-1,7 sehingga dapat disimpulkan bahwa ada dependensi *spatial* antar lokasi. Nilai $\theta(h)$ merupakan besaran nilai dependensi ekstremal antar lokasi. Koefisien ekstremal bernilai 1 menunjukkan adanya dependensi penuh, sedangkan koefisien ekstremal bernilai 2 menunjukkan tidak terindikasi dependensi *spatial* (independen penuh). Dengan demikian, plot koefisien ekstremal menunjukkan adanya unsur *spatial* pada data curah hujan di Kabupaten Ngawi.

4.2.6 Estimasi Parameter Spatial Extreme Value dengan pendekatan Copula

4.2.6.1 Transformasi Data Marginal GEV ke Copula

Berdasarkan tinjauan pustaka Bab 2 penelitian ini, analisis data menggunakan copula perlu dilakukan transformasi data terlebih dahulu yaitu dari unit data yang berdistribusi GEV ke unit margin copula. Proses transformasi ke copula mengalamai dua kali proses yaitu proses transformasi ke frechet untuk mendapatkan data yang bersifat lebih *heavytail*. Selanjutnya proses transformasi ke margin copula untuk membentuk model copula gaussian. Proses transformasi menggunakan persamaan (4.2). Transformasi melibatkan ketiga parameter GEV yang telah dihitung secara univariat menghasilkan data transformasi copula pada Lampiran 8.

4.2.6.2 Penentuan Kombinasi Model *Trend Surface* Terbaik menggunakan pendekatan Copula Gaussian

Setelah melakukan proses transformasi GEV *univariat* ke copula, proses selanjutnya adalah estimasi parameter *spatial extreme value* dengan pendekatan copula diperoleh dengan metode *maximum pairwise likelihood estimation* (MPLE). Dalam mengestimasi parameter dibutuhkan fungsi korelasi. Fungsi korelasi yang digunakan dalam penelitian ini yaitu korelasi *whittle-matern* yang merupakan fungsi korelasi untuk data *spatial* yang didasarkan pada pengukuran jarak. Estimasi masing-masing parameter menghasilkan persamaan yang tidak *close form,* sehingga digunakan metode iterasi numerik Nelder-Mead.

Pada bagian estimasi parameter *spatial extreme value* dengan pendekatan copula dihitung masing-masing parameter $\hat{\mu}$, $\hat{\sigma}$, dan $\hat{\xi}$ dengan menggunakan model *trend surface* seperti contoh pada persamaan (2.26). *Longitude* dan *latitude* merupakan variabel geografis yang menunjukkan koordinat letak suatu lokasi, dalam hal ini berfungsi sebagai variabel penjelas seperti yang terdapat pada model-model regresi pada umumnya. Pada penelitian ini, terdapat 15 kombinasi model *trend surface* dengan kombinasi variabel *longitude* dan *latitude*. Estimasi curah hujan ekstrem dilakukan menggunakan kombinasi model terbaik dari 15 kombinasi model yang ada. Suatu kombinasi model *trend surface* dikatakan terbaik dari kombinasi model *trend surface* lainnya apabila kombinasi model tersebut memiliki nilai AIC terkecil. Hasil perhitungan nilai AIC dari kombinasi model *trend surface* terdapat pada Tabel 4.6

Tabel 4.6 Kombinasi model *trend surface*

Kombinasi ke	Kombinasi Model	AIC
1	$\hat{\mu}(j)=\hat{\beta}_{\mu,0}+\hat{\beta}_{\mu,1}v(j)$ $\hat{\sigma}(j)=\hat{\beta}_{\sigma,0}+\hat{\beta}_{\sigma,1}u(j)$ $\hat{\xi}(j)=\hat{\beta}_{\xi,0}$	8356,333
2	$\hat{\mu}(j)=\hat{\beta}_{\mu,0}+\hat{\beta}_{\mu,1}v(j)$ $\hat{\sigma}(j)=\hat{\beta}_{\sigma,0}+\hat{\beta}_{\sigma,1}v(j)+\hat{\beta}_{\sigma,2}u(j)$ $\hat{\xi}(j)=\hat{\beta}_{\xi,0}$	8360,068
3	$\hat{\mu}(j)=\hat{\beta}_{\mu,0}+\hat{\beta}_{\mu,1}u(j)$ $\hat{\sigma}(j)=\hat{\beta}_{\sigma,0}+\hat{\beta}_{\sigma,1}v(j)$ $\hat{\xi}(j)=\hat{\beta}_{\xi,0}$	8415,206
4	$\hat{\mu}(j)=\hat{\beta}_{\mu,0}+\hat{\beta}_{\mu,1}u(j)$ $\hat{\sigma}(j)=\hat{\beta}_{\sigma,0}+\hat{\beta}_{\sigma,1}u(j)$ $\hat{\xi}(x)=\hat{\beta}_{\xi,0}$	8381,033
5	$\hat{\mu}(j)=\hat{\beta}_{\mu,0}+\hat{\beta}_{\mu,1}u(j)$ $\hat{\sigma}(j)=\hat{\beta}_{\sigma,0}+\hat{\beta}_{\sigma,1}v(j)+\hat{\beta}_{\sigma,2}u(j)$ $\hat{\xi}(j)=\hat{\beta}_{\xi,0}$	8387,024
6	$\hat{\mu}(j)=\hat{\beta}_{\mu,0}+\hat{\beta}_{\mu,1}v(j)+\hat{\beta}_{\mu,2}u(j)$ $\hat{\sigma}(j)=\hat{\beta}_{\sigma,0}+\hat{\beta}_{\sigma,1}v(j)+\hat{\beta}_{\sigma,2}u(j)$ $\hat{\xi}(j)=\hat{\beta}_{\xi,0}$	8361,167

Tabel 4.6 (Lanjutan)

Kombinasi ke	Kombinasi Model	AIC
7	$\hat{\mu}(j) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}v(j) + \hat{\beta}_{\mu,2}u(j)$ $\hat{\sigma}(j) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}u(j) + \hat{\beta}_{\sigma,2}v(j)$ $\hat{\xi}(j) = \hat{\beta}_{\xi,0}$	8361,167
8	$\hat{\mu}(j) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}u(j)$ $\hat{\sigma}(j) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}u(j) + \hat{\beta}_{\sigma,2}v(j)$ $\hat{\xi}(j) = \beta_{\xi,0}$	8387,024
9	$\hat{\mu}(j) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}v(j)$ $\hat{\sigma}(j) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}v(j)$ $\hat{\xi}(j) = \beta_{\xi,0}$	8358,916
10	$\hat{\mu}(j) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}v(j) + \hat{\beta}_{\mu,2}u(j)$ $\hat{\sigma}(j) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}v(j)$ $\hat{\xi}(j) = \beta_{\xi,0}$	8358,281
11	$\hat{\mu}(j) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}v(j) + \hat{\beta}_{\mu,2}u(j)$ $\hat{\sigma}(j) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}u(j)$ $\hat{\xi}(j) = \beta_{\xi,0}$	8365,167
12	$\hat{\mu}(j) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}v(j)$ $\hat{\sigma}(j) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}u(j) + \hat{\beta}_{\sigma,2}v(j)$ $\hat{\xi}(j) = \beta_{\xi,0}$	8360,068
13	$\hat{\mu}(j) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}u(j) + \hat{\beta}_{\mu,2}v(j)$ $\hat{\sigma}(j) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}v(j)$ $\hat{\xi}(j) = \beta_{\xi,0}$	8358,281
14	$\hat{\mu}(j) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}u(j) + \hat{\beta}_{\mu,2}v(j)$ $\hat{\sigma}(j) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}u(j)$ $\hat{\xi}(j) = \beta_{\xi,0}$	8365,167
15	$\mu(j) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}v(j)$ $\hat{\sigma}(j) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}u(j) + \hat{\beta}_{\sigma,1}v(j)$ $\hat{\xi}(j) = \beta_{\xi,0}$	8360,068

Berdasarkan Tabel 4.6 diketahui model yang terbaik adalah model ke-1 dengan AIC sebesar 8356,333. Dari model GEV terbaik dihitung nilai estimasi parameternya seperti pada Lampiran 15 bagian model 1. Hasil estimasi patameter tersebut kemudian dimasukkan kedalam model sehingga diperoleh persamaan model *trend surface* terbaik sebagai berikut:

$\hat{\mu}(j) = -455,090 - 68,06\, v(j)$

$\hat{\sigma}(j) = 135,571 - 0,885\, u\,(j)$

$\hat{\xi}(j) = -0,1578$

Setiap parameter yang terbentuk dari model *trend surface* dihitung *confidence interval* dari parameter tersebut meggunakan *standart* normal baku mengikuti persamaan (2.20), (2.21), (2.22), (2.23) dan (2.24). Perhitungan ini memerlukan nilai *standart error* dari $\hat{\boldsymbol{\beta}}$ melibatkan turunan kedua dari fungsi *ln likelihood* copula gaussian. Turunan kedua dari copula gaussian terdapat pada Lampiran 13, 14 dan 15. Nilai *confidence interval* menggunakan toleransi *error* $\alpha = 5\%$. Nilai *confidence interval* dari $\hat{\boldsymbol{\beta}}$ disajikan pada Tabel 4.7.

Tabel 4.7 *Confidence Interval* Estimator $\hat{\boldsymbol{\beta}}$ Parameter Model *trend surface*

Parameter	**Nilai Parameter**	***Standard Error***	***confidence interval***	
			Lower	***Upper***
$\beta_{\mu,0}$	-455,090	135,830	-721,318	-188,862
$\beta_{\mu,1}$	-68,060	18,249	-103,829	-32,291
$\beta_{\sigma,0}$	135,571	873,714	-1576,909	1848,052
$\beta_{\sigma,1}$	-0,885	7,843	-16,259	14,489
$\beta_{\xi,1}$	0,158	0,022	0,113	0,202

Estimasi parameter lokasi ($\hat{\mu}$), skala ($\hat{\sigma}$), dan parameter bentuk ($\hat{\xi}$) untuk masing-masing lokasi dapat ditentukan menggunakan persamaan *model trend surface* terbaik serta variabel *latitude* (v) dan *longitude* (u) pada masing-masing lokasi pengamatan. Nilai estimasi parameter copula untuk masing-masing lokasi Pos hujan di Kabupaten Ngawi disajikan dalam Tabel 4.8 sebagai berikut:

Tabel 4.8 Nilai estimasi parameter Copula

Pos Hujan	***Latitude***	***Longitude***	***Location*** $(\hat{\mu})$	***Scale*** $(\hat{\sigma})$	***Shape*** $(\hat{\xi})$
Gemarang	-7,396	111,366	48,268	37,057	-0,158
Guyung	-7,506	111,410	55,686	37,017	-0,158
Karangjati	-7,461	111,613	52,624	36,838	-0,158
Kedungbendo	-7,387	111,543	47,588	36,900	-0,158
Kedunggalar	-7,408	111,312	49,085	37,103	-0,158
Kendal	-7,560	111,289	59,430	37,125	-0,158
Kricak	-7,394	111,344	48,132	37,075	-0,158
Mantingan	-7,386	111,150	47,519	37,248	-0,158
Mardisari	-7,428	111,406	50,446	37,021	-0,158
Papungan	-7,383	111,369	47,383	37,053	-0,158
Paron	-7,437	111,396	51,058	37,030	-0,158

4.2.7 *Return Level* Curah Hujan Ekstrem

Nilai estimasi parameter Copula yang diperoleh digunakan untuk menghitung *return level*. *Return level* merupakan nilai estimasi curah hujan ekstrem pada periode waktu tertentu. Perhitungan *return level* copula yang merupakan nilai awal sebelum proses transformasi GEV diperoleh menggunakan persamaan (2.29), dimana *T*= 5 tahun x 4 (banyaknya blok)=20. Perhitungan *return level* copula disajikan pada Tabel 4.9

Tabel 4.9 Nilai *Return Level* copula selama 5 tahun

Pos Hujan	***Return Level***
Gemarang	136,142
Guyung	143,466
Karangjati	139,980
Kedungbendo	135,090
Kedunggalar	137,070
Kendal	147,465
Kricak	136,050
Mantingan	135,847
Mardisari	138,236
Papungan	135,249
Paron	138,869

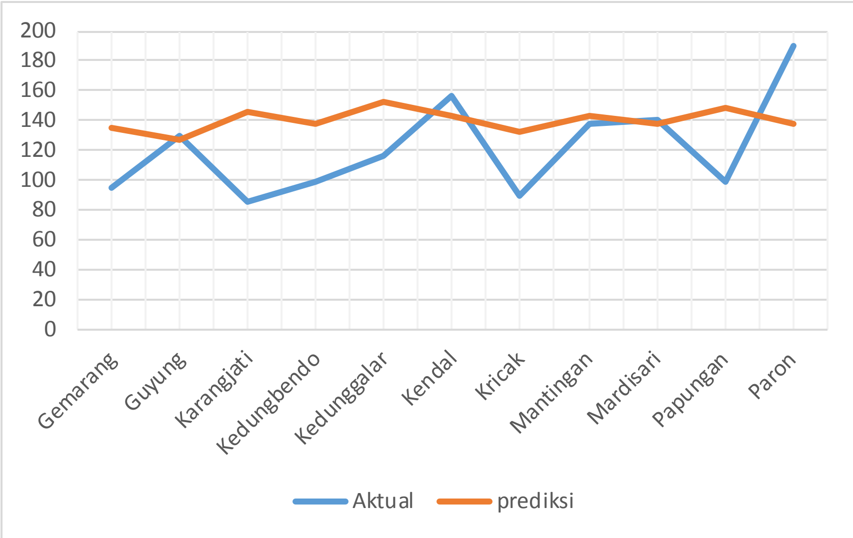
Nilai hasil *Return Level* merupakan nilai masih dalam bentuk pemodelan data curah hujan copula gaussian, oleh karena itu diperlukan nilai pengmebalian transformasi dari pemodelan copula gaussian ke GEV. Proses pengembalian transformasi melewati 2 proses yaitu pengembalian ke distribusi normal dan invers CDF disrtibusi GEV. Tabel 4.10 berisi nilai pengembalian pemodelan copula gaussian ke GEV. Prediksi nilai untuk curah hujan GEV selama 5 tahun kedepan di sebelas pos hujan disajikan pada Tabel 4.10. Nilai prediksi dibandingkan dengan nilai ekstrem aktual yang berasal dari data *testing* sehingga dapat dihitung besaran error atau kesalahan prediksi.

Tabel 4.10 Nilai prediksi *Return Level* GEV (mm/hari)

Pos Hujan	***Return Level***		**|*error*| (%)**
	Aktual	**Prediksi**	
Gemarang	95	134,603	41,688
Guyung	**130**	**127,458**	**1,955**
Karangjati	85	145,183	70,803
Kedungbendo	99	137,314	38,701
Kedunggalar	116	151,838	30,894
Kendal	**156**	**142,518**	**2,668**
Kricak	89	132,799	60,133
Mantingan	**138**	**142,616**	**3,345**
Mardisari	**141**	**138,079**	**2,072**
Papungan	99	148,070	49,566
Paron	**190**	**137,606**	**27,576**

Keterangan : Pos hujan yang di *bold* menunjukkan lokasi dengan *error* prediksi kurang dari 30% atau cukup baik.

Besaran *error* atau kesalahan prediksi dapat dihitung berdasarkan variabel prediksi dan aktual curah hujan, lalu RMSE dapat dihitung dengan menggunakan persamaan (2.30). Berdasarkan persamaan tersebut diperoleh nilai RMSE sebesar 38,155. Nilai aktual dan prediksi apabila disajikan dalam bentuk grafik dapat dilihat pada gambar 4.4 sebagai berikut



Gambar 4.4 Nilai aktual dan prediksi Curah Hujan Kabupaten Ngawi

Berdasarkan informasi BMKG, adanya perbedaan antara permalan dengan data aktual 25%-30% dianggap masih cukup baik. Berdasarkan gambar 4.4 terdapat 5 pos hujan yang nilai prediksi dan nilai aktual dibawah error 30%, antara lain Guyung, Kendal, Mantingan, Mardisari dan Paron. Enam pos Hujan lainnya mempunyai nilai prediksi dengan nilai aktual lebih dari 30%. Kondisi ini kemungkinan diakibatkan banyak faktor variabel lain yang perlu dikaji dalam penentuan prediksi curah hujan misalkan kecepatan angin, kelembaban udara sehingga perlu penelitian lebih lanjut untuk menentukan prediksi curah hujan yang nilai prediksinya mendekati nilai aktual.

Dalam penelitian sebelumnya telah dilakukan pemodelan *spatial extreme value* dengan proses *max-stable* menggunakan model smith dengan tahun data *tranining* dan *testing* yang sama pada 9 pos hujan diperoleh nilai RMSE sebesar 32,078. Sehingga dapat ditarik kesimpulan bahwa pada data curah hujan ekstrem Kabupaten Ngawi lebih baik menggunakan pemodelan max-stable proses daripada copula. Dalam penelitian sebelumnya juga telah dilakukan pemodelan *spatial extreme value* dengan copula untuk studi kasus curah hujan di Kabupaten Indramayu. Dalam penelitian tersebut hasil prediksi paling tepat ketika mem-prediksi selama 1 tahun, oleh karena itu dilakukan pula prediksi 1 tahun untuk melihat perbandingan hasil prediksi. Tabel 4.11 merupakan hasil prediksi selama 1 tahun dengan data *testing* tahun 2010-2011.

Tabel 4.11 Prediksi *Return Level* GEV selama 1 tahun

Pos Hujan	Aktual	prediksi	|error|(%)
Gemarang	87	91,002	4,601
Guyung	120	100,349	16,376
Karangjati	84	93,189	10,939
Kedungbendo	99	88,753	10,350
Kedunggalar	76	89,631	17,936
Kendal	156	103,106	42,544
Kricak	79	91,018	30,514
Mantingan	138	85,291	38,194
Mardisari	141	92,679	34,270
Papungan	97	95,264	1,789
Paron	135	93,218	30,949

Dalam Tabel 4.11 terdapat 6 Pos hujan yang nilai prediksi dan nilai aktual dibawah error 30%, dan 5 Pos Hujan lainnya mempunyai nilai prediksi dengan nilai aktual lebih dari 30%. Nilai RMSE pada tabel 4.11 adalah 33,878. Dari Tabel 4.10 dan 4.11 hasil prediksi 1 tahun memberikan hasil lebih baik daripada hasil prediksi 5 tahun, dilihat dari jumlah Pos hujan yang nilai errornya kurang dari 30% lebih banyak dan nilai RMSE lebih kecil.

BAB 5
KESIMPULAN DAN SARAN

5.1 Kesimpulan

Berdasarkan analisis yang telah dilakukan, terdapat beberapa kesimpulan diantaranya:

1. Estimasi parameter *Spatial Extreme Value* dengan pendekatan Copula dapat menggunakan MPLE. Estimasi parameter melalui MPLE, diawali dengan PDF dari copula gaussian kemudian disubtitusikan ke dalam fungsi *Pairwise Likelihood*. Selanjutnya membentuk fungsi *ln pairwise likelihood* dari fungsi *Pairwise Likelihood*. Kemudian menurunkan parameter terhadap fungsi *Pairwise Likelihood*. Estimasi parameter menghasilkan persamaan yang tidak *close form* dan selanjutnya diselesaikan dengan metode iterasi numerik Nelder-Mead.
2. Model curah hujan ekstrem Kabupaten Ngawi menggunakan estimasi parameter *Spatial Extreme Value* dengan pendekatan copula menghasilkan model *trend surface* sebagai berikut:

   $$\hat{\mu}(j) = -455,090 \text{ -}68,06\ v(j)$$

   $$\hat{\sigma}(j) = 135,571\text{-}0,885\ u\ (j)$$

   $$\hat{\xi}(j) = -0,1578$$

   Nilai RMSE sebesar 38,155. Nilai *return level* estimasi parameter memberikan hasil prediksi yaitu terdapat 5 Pos hujan yang nilai prediksi dan nilai aktual dibawah error 30%, antara lain Guyung, Kendal, Mantingan, Mardisari dan Paron. Enam Pos Hujan lainnya mempunyai nilai prediksi dengan nilai aktual lebih dari 30%.

5.2 Saran

Saran yang dapat diberikan untuk penelitian mendatang adalah

1. Perlu dilakukan penelitian *Spatial Extreme Value* dengan pendekatan Copula menggunakan Copula *student-t* untuk melihat perbandingan dari sisi validitas pada nilai RMSE dan sisi kebaikan model yaitu nilai AIC yang dihasilkan.

2. Untuk mendapatkan prediksi *return level* yang akurat, maka perlu eksplorasi terkait variabel prediktor yang akan digunakan serta menentukan model spatial GEV yang sesuai dengan data.
3. Pada penelitian selanjutnya sebaiknya dilakukan analisis *copula* menggunakan data simulasi untuk menentukan model yang sesuai sehingga menghasilkan estimasi parameter dan *return level* yang lebih baik.

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Lampiran 1. Turunan Pertama Fungsi *Ln Pairrwise Likelihood* Copula Gaussian terhadap parameter $\boldsymbol{\beta}_{\boldsymbol{\mu}}$

$$
\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \frac{\partial \left(\begin{array}{l} \displaystyle\sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \ln \left(\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} . \right. \\
\left. \left\{ \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) . \\
\left[\frac{1}{2} \left(\left(\Phi^{-1} \left(\exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right) \Phi^{-1} \left(\exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{T} . \right. \\
\left. \left(\rho(h)^{-1} \right) \cdot \left(\Phi^{-1} \left(\exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \Phi^{-1} \left(\exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right) \right] - 0.5 \ln |\rho(h)| \end{array} \right)}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}}
$$

$$
\frac{\partial \ell(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = 0
$$

$$
\begin{aligned}
0 = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} & \left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)+\right. \\
& \left.\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]+ \\
& \left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{k}}\right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)+\right. \\
& \left.\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{k}}\right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]
\end{aligned}
$$

$$\left[\frac{1}{2}\left(\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{T}\cdot\right.\right.$$

$$\left.\left.\left(\rho\left(h\right)^{-1}\right)\cdot\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\right)\right]+$$

$$\ln\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot$$

$$\frac{1}{2}\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]\right]$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\ \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}.\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{k}}\right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]\right]$$

$$\cdot\left(\rho\left(h\right)^{-1}\right)\cdot\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{T}+$$

$$\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{d_{j}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right).\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot(-\mathbf{d}_{\mathbf{j}})}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)^{T}.$$

$$\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}.\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{\mathbf{k}})}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\Bigg)$$

$$\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Bigg)\cdot\left(\rho(h)^{-1}\right)\right)$$

Lampiran 2. Turunan Pertama Fungsi *Ln Pairrwise Likelihood Copula Gaussian* terhadap parameter $\boldsymbol{\beta}_{\boldsymbol{\sigma}}$

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_{\sigma}} = \frac{\partial\left(\begin{array}{l}
\displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \ln\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right. \\
\left.\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right). \\
\left[\frac{1}{2}\left(\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\ \Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{T}.\right.\right. \\
\left.\left.\left(\rho(h)^{-1}\right)\cdot\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\ \Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\right)\right] -0.5\ln|\rho(h)|
\end{array}\right)}{\partial \boldsymbol{\beta}_{\sigma}}$$

$$\frac{\partial \ell(\boldsymbol{\beta})}{\boldsymbol{\beta}_{\boldsymbol{\sigma}}} = 0$$

$$
\begin{aligned}
0 = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \Bigg[\Bigg(& \frac{1}{\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\}} \left\{ -\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}} - 1} \right\} . \left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} + \frac{2\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} . \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}} \right) \Bigg) + \\
& \left(\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}} - 1} \right\} . \left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} \right) \right) \right) \Bigg] + \\
\Bigg[\Bigg(& \frac{1}{\left\{ \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\}} \left\{ -\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left[\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} \right) \right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}} - 1} \right\} . \left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} + \frac{2\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} . \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}} \right) \Bigg) + \\
& \left(\left\{ \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}} - 1} \right\} . \left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} \right) \right) \right) \Bigg].
\end{aligned}
$$

$$
\left[\begin{array}{l}
\frac{1}{2}\left[\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]^{T}\cdot \\
\left(\rho\left(h\right)^{-1}\right)\cdot\left[\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]
\end{array}\right]+
$$

$$
\ln\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\right.
$$

$$
\left.\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).
$$

$$
\frac{1}{2}\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right].\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}.\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)\right]^{T}
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\ \exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}.\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)\right]
$$

$$\cdot\left(\rho\left(h\right)^{-1}\right)\cdot\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{T}+$$

$$\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{j}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)^{T}$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right].$$

$$\cdot\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\left(\rho\left(h\right)^{-1}\right)\right)$$

Lampiran 3. Turunan Pertama Fungsi *Ln Pairwise Likelihood Copula Gaussian* terhadap parameter β_{ξ}

$$
\frac{\partial l(\boldsymbol{\beta})}{\partial \beta_{\xi}} = \frac{\partial\left(\begin{array}{l}
\displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \ln\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}. \right.\\
\left.\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\\
\left[\frac{1}{2}\left(\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right) \Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{T}. \right.\\
\left.\left(\rho(h)^{-1}\right)\cdot\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right) \Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\right] -0.5\ln|\rho(h)|
\end{array}\right)}{\partial \beta_{\xi}}
$$

Memisalkan $a, b,$ dan c seperti penurunan sebelumnya sehingga diperoleh

$$
\begin{aligned}
\frac{\partial l(\boldsymbol{\beta})}{\partial \beta_{\xi}} &= \frac{\partial (a \cdot b - c)}{\partial \beta_{\xi}} = 0 \\
&= \frac{\partial a}{\partial \beta_{\xi}} b + a \frac{\partial b}{\partial \beta_{\xi}} - \frac{\partial c}{\partial \beta_{\xi}} = 0
\end{aligned}
$$

$$
\frac{\partial a}{\partial \beta_{\xi}} = \left[\left(\frac{1}{\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}} \left\{ \left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \cdot \left(\ln \left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right) \right) \cdot \mathbf{d}_{\mathbf{j}} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right) +
$$

$$
\left(\left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \cdot \left(\ln \left(- \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right] \right) \right) \cdot \left(- \mathbf{d}_{\mathbf{j}} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right) \right) \Bigg] +
$$

$$\Bigg[\Bigg(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left(\ln\left(\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)\cdot\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\Bigg)+$$

$$\Bigg(\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left(\ln\left(-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg)\Bigg]$$

$$\frac{\partial b}{\partial \beta_{\xi}}=\frac{1}{2}\Bigg[\Bigg[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.$$

$$\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg]\Bigg[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Bigg)$$

$$\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_{k}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\Bigg]\cdot$$

$$\cdot\left(\rho\left(h\right)^{-1}\right)\cdot\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Bigg)^{T}\right)$$

$$\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.$$

$$\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\Bigg]\Bigg]\Bigg)\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.$$

$$\left[\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_{k}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot x_{ki}\right]^{T}\cdot\left(\rho\left(h\right)^{-1}\right).$$

$$\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\Bigg)\Bigg)$$

$$\frac{\partial c}{\partial \beta_{\xi}} = 0$$

$$\frac{\partial \ell(\boldsymbol{\beta})}{\beta_{\xi}} = 0$$

$$0 = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)\cdot\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right) +$$

$$\left(\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left(-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)\Bigg] +$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left(\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)\cdot\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)+\right.$$

$$\left.\left(\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left(-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]\right)\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right]$$

$$\left[\frac{1}{2}\left(\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\ \Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\right)^{T}\cdot\right.$$
$$\left.\left(\rho\left(h\right)^{-1}\right)\cdot\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\right]+$$

$$\ln\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).$$

$$\frac{1}{2}\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.\right.$$

$$\left.\left.\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right]\right]\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\right.$$

$$\left.\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{k}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right]\cdot$$

$$\cdot\left(\rho\left(h\right)^{-1}\right).\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\ \Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{T}\right)$$

$$\Bigg(\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg)\Bigg).\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.$$

$$\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg]\Bigg]\Bigg)\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg)\Bigg).\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}.$$

$$\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{k}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}\Bigg]^{T}\cdot\left(\rho\left(h\right)^{-1}\right).$$

$$\Bigg(\Phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg)\ \Phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg)\Bigg)\Bigg)\Bigg)$$

Lampiran 4. Data 11 Pos Hujan

Observasi ke-	Tahun	Bulan	Hari ke-	Curah Hujan								
				gemarang	guyung	karangjati	kedungbendo	Kedunggalar	kendal	kricak	mantingan	mardisari
1	1990	12	1	17	3	7	0	0	0	0	0	0
2	1990	12	2	14	25	0	0	0	58	21	14	0
3	1990	12	3	21	32	70	0	0	16	14	20	0
4	1990	12	4	18	36	18	0	0	53	59	9	0
5	1990	12	5	22	18	15	0	0	12	32	31	0
6	1990	12	6	19	6	6	0	0	13	50	4	0
7	1990	12	7	0	0	0	0	0	18	0	0	0
8	1990	12	8	0	3	30	0	0	10	16	0	0
9	1990	12	9	0	2	5	0	0	25	8	0	0
10	1990	12	10	0	1	20	0	0	8	14	0	0
11	1990	12	11	16	22	0	0	0	0	18	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
5446	2005	10	26	0	0	0	0	0	0	0	0	0
5447	2005	10	27	0	0	0	0	0	0	0	0	0
5448	2005	10	28	0	6	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
9129	2015	11	28	0	0	0	0	0	0	0	0	0
9130	2015	11	29	0	0	0	0	0	0	0	0	0
9131	2015	11	30	0	0	0	0	0	0	0	0	4

Lampiran 4. (Lanjutan)

Observasi ke-	Tahun	Bulan	Hari ke-	Curah Hujan	
				papungan	paron
1	1990	12	1	0	0
2	1990	12	2	0	5
3	1990	12	3	0	40
4	1990	12	4	0	58
5	1990	12	5	0	8
6	1990	12	6	0	0
7	1990	12	7	0	0
8	1990	12	8	0	0
9	1990	12	9	0	0
10	1990	12	10	0	6
11	1990	12	11	0	0
⋮	⋮	⋮	⋮	⋮	⋮
5446	2005	10	26	0	0
5447	2005	10	27	0	10
5448	2005	10	28	0	0
⋮	⋮	⋮	⋮	⋮	⋮
9129	2015	11	28	0	0
9130	2015	11	29	0	0
9131	2015	11	30	0	3

Lampiran 5. Data *Training*

Blok ke-	Peri-ode	Tahun	Curah Hujan								
			gemarang	guyung	karangjati	kedungbendo	Kedunggalar	kendal	kricak	mantingan	mardisari
1	DJF	1990-1991	82	86	70	65	23	71	67	115	54
2	MAM	1991	54	72	112	70	63	78	59	87	82
3	JJA	1991	0	0	0	0	0	0	0	5	0
4	SON	1991	86	68	57	100	39	51	45	69	85
5	DJF	1991-1992	76	82	87	108	43	155	96	92	115
6	MAM	1992	56	0	96	58	58	96	69	105	46
7	JJA	1992	36	0	51	54	35	124	34	45	39
8	SON	1992	95	0	70	105	34	80	98	69	85
9	DJF	1992-1993	131	89	126	90	45	116	87	121	128
10	MAM	1993	24	71	60	63	9	95	70	71	47
11	JJA	1993	75	53	54	29	43	88	50	55	58
12	SON	1993	23	27	6	106	5	7	27	26	12
13	DJF	1993-1994	61	97	153	107	129	71	63	67	75
14	MAM	1994	82	136	90	84	36	114	54	59	81
15	JJA	1994	5	0	0	0	0	0	0	22	0
16	SON	1994	55	51	42	33	42	52	55	32	62
17	DJF	1994-1995	109	47	108	88	53	109	98	121	73
18	MAM	1995	85	112	126	50	65	104	96	53	78
19	JJA	1995	135	40	54	80	17	25	81	21	47
20	SON	1995	90	78	97	70	42	89	138	115	62
21	DJF	1995-1996	115	94	95	106	65	66	95	82	94
22	MAM	1996	137	117	44	64	35	114	125	77	162
23	JJA	1996	23	36	46	27	17	83	12	69	26
24	SON	1996	99	107	62	55	25	86	61	42	67
25	DJF	1996-1997	120	37	76	178	36	143	85	153	130
26	MAM	1997	100	87	37	37	28	64	78	71	71
27	JJA	1997	98	0	12	127	10	93	41	31	58

Lampiran 5. (Lanjutan)

Blok ke-	Peri-ode	Tahun	Curah Hujan								
			gemarang	guyung	karangjati	kedungbendo	Kedunggalar	kendal	kricak	mantingan	mardisari
28	SON	1997	55	76	42	18	17	20	45	67	37
29	DJF	1997-1998	132	97	118	91	65	82	116	97	99
30	MAM	1998	58	0	145	40	30	89	82	65	48
31	JJA	1998	70	0	145	46	30	89	82	65	55
32	SON	1998	86	54	70	88	68	47	97	80	75
33	DJF	1998-1999	94	106	67	57	73	82	90	150	103
34	MAM	1999	68	98	92	24	78	76	80	150	58
35	JJA	1999	30	82	35	23	28	26	6	180	40
36	SON	1999	98	97	201	25	87	60	84	65	77
37	DJF	1999-2000	118	71	81	58	49	97	115	108	105
38	MAM	2000	92	80	107	79	108	92	98	240	90
39	JJA	2000	30	97	67	8	65	26	18	30	36
40	SON	2000	80	37	131	76	78	100	90	77	90
41	DJF	2000-2001	120	89	141	24	118	80	62	65	80
42	MAM	2001	95	94	87	36	114	90	80	125	100
43	JJA	2001	54	41	61	53	97	0	97	109	72
44	SON	2001	149	77	136	65	97	38	37	65	90
45	DJF	2001-2002	90	66	143	59	115	60	125	90	105
46	MAM	2002	80	65	85	34	80	95	49	47	58
47	JJA	2002	17	43	0	34	0	14	0	0	0
48	SON	2002	90	57	73	32	50	85	98	82	76
49	DJF	2002-2003	90	125	89	68	75	97	98	79	131
50	MAM	2003	80	49	39	48	40	115	68	79	108
51	JJA	2003	0	54	16	7	5	13	27	15	0

Lampiran 5. (Lanjutan)

Blok ke-	Peri-ode	Tahun	Curah Hujan								
			gemarang	guyung	karangjati	kedungbendo	Kedunggalar	kendal	kricak	mantingan	mardisari
52	SON	2003	0	98	60	88	91	70	97	58	101
53	DJF	2003-2004	57	95	60	40	61	113	90	100	86
54	MAM	2004	94	39	56	59	54	97	20	80	158
55	JJA	2004	0	29	0	41	0	38	40	25	12
56	SON	2004	19	0	0	23	0	18	0	25	23
57	DJF	2004-2005	100	124	76	87	83	105	92	98	128
58	MAM	2005	60	121	135	67	98	82	95	53	71
59	JJA	2005	95	98	65	85	17	54	49	40	38
60	SON	2005	54	99	65	60	89	28	62	20	104
61	DJF	2005-2006	95	116	100	60	94	50	70	80	130
62	MAM	2006	25	101	80	71	97	69	0	70	59
63	JJA	2006	0	0	0	0	0	7	0	0	12
64	SON	2006	60	47	0	18	14	32	0	50	21
65	DJF	2006-2007	75	60	80	60	51	66	50	80	80
66	MAM	2007	96	122	63	81	48	123	60	110	68
67	JJA	2007	0	80	40	12	13	24	0	0	30
68	SON	2007	36	62	50	86	46	133	76	70	90
69	DJF	2007-2008	160	101	70	160	125	87	150	75	115
70	MAM	2008	45	107	65	105	87	103	32	80	67
71	JJA	2008	0	63	0	23	0	0	25	80	47
72	SON	2008	45	122	80	24	71	84	28	55	105
73	DJF	2008-2009	40	105	80	57	111	97	73	81	85
74	MAM	2009	85	91	85	86	70	145	98	75	138
75	JJA	2009	20	10	85	22	65	80	21	0	15

Lampiran 5. (Lanjutan)

Blok ke-	Peri-ode	Tahun	Curah Hujan								
			gemarang	guyung	karangjati	kedungbendo	Kedunggalar	kendal	kricak	mantingan	mardisari
76	SON	2009	40	120	70	73	28	100	42	58	72
77	DJF	2009-2010	78	118	85	67	64	158	70	61	110
78	MAM	2010	80	118	86	91	99	117	70	90	140
79	JJA	2010	40	18	56	31	42	57	30	34	87

Blok ke-	Periode	Tahun	Curah Hujan	
			papungan	paron
1	DJF	1990-1991	100	58
2	MAM	1991	52	95
3	JJA	1991	0	0
4	SON	1991	0	97
5	DJF	1991-1992	75	118
6	MAM	1992	80	67
7	JJA	1992	50	55
8	SON	1992	75	81
9	DJF	1992-1993	92	122
10	MAM	1993	21	59
11	JJA	1993	70	48
12	SON	1993	12	11
13	DJF	1993-1994	93	81
14	MAM	1994	96	72

Blok ke-	Periode	Tahun	Curah Hujan	
			papungan	papungan
15	JJA	1994	0	0
16	SON	1994	88	80
17	DJF	1994-1995	97	81
18	MAM	1995	86	47
19	JJA	1995	40	37
20	SON	1995	59	40
21	DJF	1995-1996	100	59
22	MAM	1996	150	150
23	JJA	1996	47	31
24	SON	1996	45	85
25	DJF	1996-1997	193	127
26	MAM	1997	95	86
27	JJA	1997	30	56
28	SON	1997	45	41
29	DJF	1997-1998	116	96
30	MAM	1998	71	49
31	JJA	1998	122	72
32	SON	1998	80	115
33	DJF	1998-1999	81	92
34	MAM	1999	79	60
35	JJA	1999	20	45
36	SON	1999	84	65
37	DJF	1999-2000	65	111
38	MAM	2000	60	93

Blok ke-	Periode	Tahun	Curah Hujan	
			papungan	paron
39	JJA	2000	35	51
40	SON	2000	60	110
41	DJF	2000-2001	92	41
42	MAM	2001	75	45
43	JJA	2001	37	0
44	SON	2001	101	0
45	DJF	2001-2002	76	0
46	MAM	2002	52	45
47	JJA	2002	0	0
48	SON	2002	115	78
49	DJF	2002-2003	115	133
50	MAM	2003	63	105
51	JJA	2003	39	3
52	SON	2003	91	97
53	DJF	2003-2004	74	87
54	MAM	2004	95	161
55	JJA	2004	0	8
56	SON	2004	38	20
57	DJF	2004-2005	99	126
58	MAM	2005	98	74
59	JJA	2005	95	34
60	SON	2005	54	101
61	DJF	2005-2006	95	129
62	MAM	2006	57	58
63	JJA	2006	0	14

Blok ke-	Peri-ode	Tahun	Curah Hujan	
			papungan	paron
64	SON	2006	48	18
65	DJF	2006-2007	73	79
66	MAM	2007	98	65
67	JJA	2007	4	32
68	SON	2007	74	88
69	DJF	2007-2008	163	115
70	MAM	2008	61	67
71	JJA	2008	0	48
72	SON	2008	57	105
73	DJF	2008-2009	80	71
74	MAM	2009	128	81
75	JJA	2009	25	12
76	SON	2009	27	61
77	DJF	2009-2010	89	96
78	MAM	2010	95	130
79	JJA	2010	35	60

Lampiran 6. Data *Testing*

Blok ke-	Peri-ode	Tahun	Curah Hujan								
			gemarang	guyung	karangjati	kedungbendo	Kedunggalar	kendal	kricak	mantingan	mardisari
1	SON	2010	87	120	82	95	76	98	68	138	77
2	DJF	2010-2011	75	83	84	98	47	156	35	87	95
3	MAM	2011	70	78	74	99	76	90	79	64	141
4	JJA	2011	82	99	0	0	31	63	30	65	30
5	SON	2011	80	124	64	75	92	94	83	68	80
6	DJF	2011-2012	70	114	72	98	61	119	58	126	85
7	MAM	2012	73	123	51	59	57	66	48	41	100
8	JJA	2012	25	26	4	0	17	30	22	42	35
9	SON	2012	40	57	23	55	75	67	46	55	64
10	DJF	2012-2013	80	96	69	93	63	93	58	90	130
11	MAM	2013	95	108	62	97	94	102	82	96	85
12	JJA	2013	45	128	60	65	60	78	48	23	39
13	SON	2013	41	84	51	55	71	66	58	56	44
14	DJF	2013-2014	75	115	76	99	65	34	61	93	68
15	MAM	2014	57	130	66	80	57	45	67	44	55
16	JJA	2014	29	40	20	41	12	86	28	47	13
17	SON	2014	27	45	50	17	43	35	56	72	79
18	DJF	2014-2015	60	120	78	99	39	70	68	60	83
19	MAM	2015	59	55	85	79	116	66	89	94	135
20	JJA	2015	13	7	23	0	5	64	8	0	0
21	SON	2015	36	25	25	0	46	60	49	22	45

Blok ke-	Peri-ode	Tahun	Curah Hujan	
			papungan	paron
1	SON	2010	97	65
2	DJF	2010-2011	86	96
3	MAM	2011	73	135
4	JJA	2011	92	43
5	SON	2011	85	88
6	DJF	2011-2012	99	82
7	MAM	2012	63	105
8	JJA	2012	38	30
9	SON	2012	45	39
10	DJF	2012-2013	71	125
11	MAM	2013	98	98
12	JJA	2013	40	27
13	SON	2013	40	48
14	DJF	2013-2014	49	49
15	MAM	2014	58	72
16	JJA	2014	27	28
17	SON	2014	42	97
18	DJF	2014-2015	59	85
19	MAM	2015	58	190
20	JJA	2015	0	0
21	SON	2015	26	47

Lampiran 7. Tabel Anderson Darling

n	α							
	0,250	0,150	0,100	0,050	0,025	0,010	0,005	0,001
10	1,2419	1,6277	1,9518	2,5121	3,0990	3,9083	4,5175	5,9897
20	1,2500	1,6290	1,9385	2,5020	3,0731	3,8995	4,5117	5,9852
30	1,2457	1,6210	1,9313	2,5130	3,1111	3,9673	4,5309	5,8924
40	1,2450	1,6173	1,9362	2,5042	3,1047	3,9397	4,5889	6,1275
50	1,2425	1,6163	1,9277	2,4941	3,0933	3,9200	4,5211	5,943
60	1,2464	1,6225	1,9367	2,5044	3,0776	3,9234	4,4858	6,0808
70	1,2515	1,6245	1,9304	2,4959	3,0889	3,8673	4,5326	5,9428
80	1,2384	1,6148	1,9235	2,4951	3,0778	3,8458	4,4808	5,9249
90	1,2461	1,6177	1,9326	2,5064	3,1020	3,9239	4,5856	6,0412
100	1,2399	1,6235	1,9235	2,4901	3,0655	3,8319	4,4068	5,8987
Mean	1,2453	1,6211	1,9355	2,4986	3,0916	3,9033	4,5416	6,0255

Lampiran 8. Trasnsformasi ke copula (u)

Blok	Gemarang	Guyung	Karangjati	Kedungbendo	Kedunggalar	Kendal	Kricak	Mantingan	Mardisari	Papungan	Paron
1	0,627	0,616	0,502	0,197	0,590	0,406	0,541	0,853	0,315	0,742	0,411
2	0,356	0,472	0,826	0,636	0,642	0,438	0,457	0,682	0,594	0,314	0,756
3	0,040	0,053	0,031	0,051	0,025	0,031	0,042	0,025	0,024	0,028	0,039
4	0,664	0,433	0,380	0,363	0,872	0,262	0,315	0,521	0,624	0,028	0,771
5	0,569	0,574	0,653	0,409	0,908	0,975	0,811	0,720	0,861	0,530	0,899
6	0,375	0,053	0,724	0,582	0,511	0,700	0,562	0,803	0,244	0,576	0,500
7	0,206	0,053	0,325	0,318	0,465	0,925	0,218	0,277	0,190	0,296	0,382
8	0,741	0,053	0,502	0,308	0,895	0,463	0,826	0,521	0,624	0,530	0,636
9	0,946	0,647	0,891	0,433	0,812	0,888	0,738	0,877	0,924	0,680	0,916
10	0,130	0,462	0,408	0,093	0,568	0,681	0,573	0,541	0,253	0,096	0,421
11	0,560	0,302	0,352	0,409	0,190	0,588	0,364	0,378	0,353	0,483	0,316
12	0,125	0,141	0,046	0,072	0,900	0,069	0,166	0,118	0,052	0,060	0,073
13	0,422	0,730	0,964	0,983	0,904	0,406	0,499	0,501	0,524	0,688	0,636
14	0,627	0,997	0,678	0,330	0,769	0,856	0,404	0,420	0,585	0,712	0,549
15	0,053	0,053	0,031	0,051	0,025	0,031	0,042	0,093	0,024	0,028	0,039
16	0,365	0,287	0,247	0,398	0,229	0,275	0,415	0,161	0,392	0,647	0,626
17	0,843	0,258	0,803	0,525	0,798	0,825	0,826	0,877	0,504	0,720	0,636
18	0,655	0,870	0,891	0,656	0,418	0,800	0,811	0,358	0,555	0,630	0,307
19	0,958	0,211	0,352	0,147	0,736	0,150	0,683	0,087	0,253	0,214	0,223
20	0,699	0,532	0,731	0,398	0,642	0,612	0,986	0,853	0,392	0,378	0,247
21	0,878	0,699	0,716	0,656	0,900	0,356	0,804	0,641	0,707	0,742	0,421
22	0,963	0,909	0,264	0,318	0,579	0,856	0,959	0,597	0,994	0,961	0,986
23	0,125	0,187	0,281	0,147	0,172	0,525	0,083	0,521	0,109	0,271	0,179

Blok	Gemarang	Guyung	Karangjati	Kedungbendo	Kedunggalar	Kendal	Kricak	Mantingan	Mardisari	Papungan	Paron
24	0,773	0,827	0,427	0,216	0,477	0,562	0,478	0,248	0,443	0,254	0,672
25	0,903	0,193	0,558	0,330	0,999	0,950	0,720	0,957	0,932	0,998	0,935
26	0,781	0,626	0,208	0,245	0,270	0,337	0,654	0,541	0,483	0,704	0,681
27	0,765	0,053	0,066	0,099	0,962	0,663	0,278	0,154	0,353	0,145	0,391
28	0,365	0,512	0,247	0,147	0,103	0,125	0,315	0,501	0,175	0,254	0,255
29	0,950	0,730	0,856	0,656	0,819	0,500	0,928	0,755	0,749	0,843	0,764
30	0,394	0,053	0,948	0,265	0,303	0,612	0,693	0,481	0,261	0,492	0,325
31	0,511	0,053	0,948	0,265	0,371	0,612	0,693	0,481	0,324	0,873	0,549
32	0,664	0,310	0,502	0,686	0,798	0,238	0,819	0,624	0,524	0,576	0,885
33	0,733	0,818	0,474	0,733	0,500	0,500	0,764	0,953	0,781	0,585	0,732
34	0,491	0,740	0,693	0,775	0,147	0,425	0,673	0,953	0,353	0,567	0,430
35	0,166	0,574	0,193	0,245	0,139	0,169	0,060	0,984	0,197	0,091	0,289
36	0,765	0,730	0,998	0,840	0,155	0,319	0,711	0,481	0,544	0,612	0,480
37	0,893	0,462	0,602	0,479	0,511	0,731	0,923	0,819	0,795	0,435	0,863
38	0,716	0,553	0,797	0,939	0,727	0,650	0,826	0,999	0,671	0,388	0,740
39	0,166	0,730	0,474	0,656	0,051	0,169	0,112	0,146	0,168	0,178	0,344
40	0,608	0,193	0,909	0,775	0,700	0,769	0,764	0,597	0,671	0,388	0,858
41	0,903	0,647	0,939	0,965	0,147	0,463	0,488	0,481	0,575	0,680	0,255
42	0,741	0,699	0,653	0,956	0,260	0,638	0,673	0,892	0,757	0,530	0,289
43	0,356	0,217	0,417	0,896	0,453	0,031	0,819	0,824	0,494	0,192	0,039
44	0,986	0,522	0,925	0,896	0,590	0,219	0,243	0,481	0,671	0,749	0,039
45	0,699	0,414	0,944	0,959	0,523	0,319	0,959	0,705	0,795	0,539	0,039
46	0,608	0,405	0,637	0,791	0,239	0,681	0,354	0,297	0,353	0,314	0,289
47	0,096	0,230	0,031	0,051	0,239	0,100	0,042	0,015	0,024	0,028	0,039

Blok	Gemarang	Guyung	Karangjati	Kedungbendo	Kedunggalar	Kendal	Kricak	Mantingan	Mardisari	Papungan	Paron
48	0,699	0,335	0,530	0,491	0,219	0,550	0,826	0,641	0,534	0,837	0,608
49	0,699	0,959	0,670	0,750	0,622	0,731	0,826	0,615	0,935	0,837	0,954
50	0,608	0,272	0,223	0,375	0,395	0,875	0,552	0,615	0,817	0,416	0,827
51	0,040	0,310	0,082	0,072	0,047	0,087	0,166	0,058	0,024	0,207	0,047
52	0,040	0,740	0,408	0,865	0,798	0,387	0,819	0,409	0,765	0,672	0,771
53	0,384	0,709	0,408	0,614	0,303	0,838	0,764	0,774	0,633	0,520	0,690
54	0,733	0,205	0,370	0,537	0,523	0,731	0,123	0,624	0,991	0,704	0,995
55	0,040	0,150	0,031	0,051	0,314	0,219	0,269	0,111	0,052	0,028	0,062
56	0,105	0,053	0,031	0,051	0,139	0,113	0,042	0,111	0,094	0,199	0,113
57	0,781	0,954	0,558	0,813	0,791	0,813	0,780	0,761	0,924	0,735	0,932
58	0,413	0,936	0,922	0,901	0,611	0,500	0,804	0,358	0,483	0,727	0,569
59	0,741	0,740	0,455	0,147	0,776	0,287	0,354	0,230	0,182	0,704	0,200
60	0,356	0,750	0,455	0,853	0,534	0,188	0,488	0,082	0,788	0,332	0,801
61	0,741	0,902	0,752	0,881	0,534	0,250	0,573	0,624	0,932	0,704	0,942
62	0,136	0,770	0,593	0,896	0,652	0,375	0,042	0,531	0,363	0,360	0,411
63	0,040	0,053	0,031	0,051	0,025	0,069	0,042	0,015	0,052	0,028	0,085
64	0,413	0,258	0,031	0,125	0,103	0,200	0,042	0,327	0,085	0,279	0,103
65	0,560	0,360	0,593	0,502	0,534	0,356	0,364	0,624	0,575	0,511	0,617
66	0,750	0,943	0,436	0,468	0,744	0,913	0,467	0,829	0,453	0,727	0,480
67	0,040	0,553	0,231	0,118	0,069	0,138	0,042	0,015	0,131	0,037	0,186
68	0,206	0,378	0,316	0,444	0,784	0,938	0,634	0,531	0,671	0,520	0,699
69	0,996	0,770	0,502	0,978	0,995	0,575	0,997	0,579	0,861	0,981	0,885
70	0,276	0,827	0,455	0,840	0,895	0,787	0,203	0,624	0,443	0,397	0,500
71	0,040	0,387	0,031	0,051	0,139	0,031	0,153	0,624	0,253	0,028	0,316

Blok	Gemarang	Guyung	Karangjati	Kedungbendo	Kedunggalar	Kendal	Kricak	Mantingan	Mardisari	Papungan	Paron
72	0,276	0,943	0,593	0,715	0,147	0,537	0,173	0,378	0,795	0,360	0,827
73	0,236	0,808	0,593	0,948	0,500	0,731	0,604	0,632	0,624	0,576	0,540
74	0,655	0,668	0,637	0,705	0,784	0,962	0,826	0,579	0,957	0,898	0,636
75	0,110	0,078	0,637	0,656	0,131	0,463	0,128	0,015	0,062	0,116	0,077
76	0,236	0,930	0,502	0,245	0,672	0,769	0,287	0,409	0,494	0,127	0,440
77	0,589	0,916	0,637	0,646	0,611	0,988	0,573	0,440	0,830	0,655	0,764
78	0,608	0,916	0,645	0,905	0,819	0,900	0,573	0,705	0,962	0,704	0,945
79	0,236	0,104	0,370	0,398	0,209	0,300	0,188	0,177	0,643	0,178	0,430

Lampiran 9. Fungsi Disrtibusi curah hujan ekstrem 11 Pos hujan Kabupaten Ngawi

Pos Hujan	Fungsi Distribusi
Gemarang	$F_{\text{Gemarang}}(x) = exp\left\{-\left(1+\left(-0{,}305\frac{x-55{,}281}{39{,}331}\right)\right)^{\frac{1}{0{,}305}}\right\}$
Guyung	$F_{\text{Guyung}}(x) = exp\left\{-\left(1+\left(-0{,}534\frac{x-60{,}876}{41{,}886}\right)\right)^{\frac{1}{0{,}534}}\right\}$
Karangjati	$F_{\text{Karangjati}}(x) = exp\left\{-\left(1+\left(-0{,}190\frac{x-55{,}731}{39{,}663}\right)\right)^{\frac{1}{0{,}190}}\right\}$
Kedungbendo	$F_{\text{Kedungbendo}}(x) = exp\left\{-\left(1+\left(-0{,}200\frac{x-39{,}402}{32{,}248}\right)\right)^{\frac{1}{0{,}200}}\right\}$
Kedunggalar	$F_{\text{Kedunggalar}}(x) = exp\left\{-\left(1+\left(-0{,}097\frac{x-44{,}910}{30{,}349}\right)\right)^{\frac{1}{0{,}097}}\right\}$
Kendal	$F_{\text{Kendal}}(x) = exp\left\{-\left(1+\left(-0{,}349\frac{x-62{,}065}{39{,}861}\right)\right)^{\frac{1}{0{,}349}}\right\}$
Kricak	$F_{\text{Kricak}}(x) = exp\left\{-\left(1+\left(-0{,}304\frac{x-50{,}411}{36{,}597}\right)\right)^{\frac{1}{0{,}304}}\right\}$
Mantingan	$F_{\textbf{Mantingan}}(x) = exp\left\{-\left(1+\left(-0{,}080\frac{x-53{,}988}{35{,}691}\right)\right)^{\frac{1}{0{,}080}}\right\}$
Mardisari	$F_{\textbf{Mardisari}}(x) = exp\left\{-\left(1+\left(-0{,}278\frac{x-59{,}540}{37{,}578}\right)\right)^{\frac{1}{0{,}278}}\right\}$
Papungan	$F_{\textbf{Papungan}}(x) = exp\left\{-\left(1+\left(-0{,}214\frac{x-57{,}900}{39{,}524}\right)\right)^{\frac{1}{0{,}214}}\right\}$
Paron	$F_{\textbf{Paron}}(x) = exp\left\{-\left(1+\left(-0{,}276\frac{x-53{,}568}{38{,}550}\right)\right)^{\frac{1}{0{,}276}}\right\}$

Lampiran 10 *Syntax Software* R

```
#Package yang harus diinstall
1.  extreme
2.  nsrfa
3.  spatialextrem

#input koordinat lokasi
x<-read.table("D:/analisis data tesis/koordinat.txt", header=T)
x<-as.matrix(x)
loc=x
colnames(loc)<-c("lat", "lon")
print(loc)

#blok 3 bulan
b1<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/gemarang.txt",header=TRUE))
b2<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/guyung.txt",header=TRUE))
b3<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/karangjati.txt",header=TRUE))
b4<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/kedungbendo.txt",header=TRUE))
b5<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/kedunggalar.txt",header=TRUE))
b6<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/kendal.txt",header=TRUE))
b7<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/kricak.txt",header=TRUE))
b8<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/mantingan.txt",header=TRUE))
b9<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/mardisari.txt",header=TRUE))
b10<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/papungan.txt",header=TRUE))
b11<-as.matrix(read.table("D:/analisis data
tesis/0912/BM/paron.txt",header=TRUE))
B=matrix(c(b1,b2,b3,b4,b5,b6,b7,b8,b9,b10,b11),ncol=11)
colnames(B)=c("GEMARANG","GUYUNG","KARANGJATI","KEDUNGBENDO","KEDU
NGGALAR","KENDAL","KRICAK","MANTINGAN","MARDISARI","PAPUNGAN","PAR
ON")
print(B)

#Uji Anderson Darling

F1= F.GEV(b1, 55.28092,39.33090,-0.30452)
A1=A2(sort(F.GEV(b1, 55.28092,39.33090,-0.30452)))
AD1=A2_GOFlaio(b1, dist="GEV")
print(AD1)
F2= F.GEV(b2, 60.87602,41.88608,-0.53381)
A2=A2(sort(F.GEV(b2, 60.87602,41.88608,-0.53381)))
AD2=A2_GOFlaio(b2, dist="GEV")
print(AD2)
F3= F.GEV(b3, 55.73079,39.66276,-0.19016)
A3=A2(sort(F.GEV(b3, 55.73079,39.66276,-0.19016)))
AD3=A2_GOFlaio(b3, dist="GEV")
print(AD3)
```

```
F4= F.GEV(b4, 39.40173,32.24765,-0.19991)
A4=A2(sort(F.GEV(b4, 39.40173,32.24765,-0.19991)))
AD4=A2_GOFlaio(b4, dist="GEV")
print(AD4)
F5= F.GEV(b5, 44.98968,30.36494,-0.09754))
A5=A2(sort(F.GEV(b5, 44.98968,30.36494,-0.09754)))
AD5=A2_GOFlaio(b5, dist="GEV")
print(AD5)
F6= F.GEV(b6, 62.06489,39.86060,-0,34936)
A6=A2(sort(F.GEV(b6, 62.06489,39.86060,-0,34936)))
AD6=A2_GOFlaio(b6, dist="GEV")
print(AD6)
F7= F.GEV(b7, 50.41075,36.59654,-0.30406)
A7=A2(sort(F.GEV(b7, 50.41075,36.59654,-0.30406)))
AD7=A2_GOFlaio(b7, dist="GEV")
print(AD7)
F8= F.GEV(b8, 53.98772,35.69147,-0.07981)
A8=A2(sort(F.GEV(b8, 53.98772,35.69147,-0.07981)))
AD8=A2_GOFlaio(b8, dist="GEV")
print(AD8)
F9= F.GEV(b9, 59.54021,37.57842,-0.27829)
A9=A2(sort(F.GEV(b9, 59.54021,37.57842,-0.27829)))
AD9=A2_GOFlaio(b9, dist="GEV")
print(AD9)
F10= F.GEV(b10, 57.9003,39.52390,-0.21433)
A10=A2(sort(F.GEV(b10, 57.90030,39.52390,-0.21433)))
AD10=A2_GOFlaio(b10, dist="GEV")
print(AD10)
F11= F.GEV(b11, 53.56826,38.54955,-0.27580)
A11=A2(sort(F.GEV(b11, 53.56826,38.54955,-0.27580)))
AD11=A2_GOFlaio(b11, dist="GEV")
print(AD11)

#transformasi ke copula
z1 <- gev2frech(b1, 55.28092,39.33090,-0.30452)
z2 <- gev2frech(b2, 60.87602,41.88608,-0.53381)
z3 <- gev2frech(b3, 55.73079,39.66276,-0.19016)
z4 <- gev2frech(b4, 39.40173,32.24765,-0.19991)
z5 <- gev2frech(b5, 44.98968,30.36494,-0.09754)
z6 <- gev2frech(b6, 62.06489,39.86060,-0,34936)
z7 <- gev2frech(b7, 50.41075,36.59654,-0.30406)
z8 <- gev2frech(b8, 53.98772,35.69147,-0.07981)
z9 <- gev2frech(b9, 59.54021,37.57842,-0.27829)
z10 <- gev2frech(b10, 57.9003,39.52390,-0.21433)
z11 <- gev2frech(b11, 53.56826,38.54955,-0.27580)
Z=matrix(c(z1,z2,z3,z4,z5,z6,z7,z8,z9,z10,z11),ncol=11)
colnames(Z)=c("GEMARANG","GUYUNG","KARANGJATI","KEDUNGBENDO","KEDU
NGGALAR","KENDAL","KRICAK","MANTINGAN","MARDISARI","PAPUNGAN","PAR
ON")
print(Z)
u1<- exp(-1/z1)
u2<- exp(-1/z2)
u3<- exp(-1/z3)
u4<- exp(-1/z4)
u5<- exp(-1/z5)
u6<- exp(-1/z6)
```

```
u7<- exp(-1/z7)
u8<- exp(-1/z8)
u9<- exp(-1/z9)
u10<- exp(-1/z10)
u11<- exp(-1/z11)
U=matrix(c(u1,u2,u3,u4,u5,u6,u7,u8,u9,u10,u11),ncol=11)
#perhitungan Koefisien ekstermal
fitextcoeff(U, loc, estim = "Smith")
#model copula gaussian
q1<- as.matrix(read.table("D:/analisis data
tesis/C/u1.txt",header=TRUE))
q2<- as.matrix(read.table("D:/analisis data
tesis/C/u2.txt",header=TRUE))
q3<- as.matrix(read.table("D:/analisis data
tesis/C/u3.txt",header=TRUE))
q4<- as.matrix(read.table("D:/analisis data
tesis/C/u4.txt",header=TRUE))
q5<- as.matrix(read.table("D:/analisis data
tesis/C/u5.txt",header=TRUE))
q6<- as.matrix(read.table("D:/analisis data
tesis/C/u6.txt",header=TRUE))
q7<- as.matrix(read.table("D:/analisis data
tesis/C/u7.txt",header=TRUE))
q8<- as.matrix(read.table("D:/analisis data
tesis/C/u8.txt",header=TRUE))
q9<- as.matrix(read.table("D:/analisis data
tesis/C/u9.txt",header=TRUE))
q10<- as.matrix(read.table("D:/analisis data
tesis/C/u10.txt",header=TRUE))
q11<- as.matrix(read.table("D:/analisis data
tesis/C/u11.txt",header=TRUE))
Q=matrix(c(q1,q2,q3,q4,q5,q6,q7,q8,q9,q10,q11),ncol=11)
loc.form <- q ~ lat
scale.form <- q ~ lon
shape.form <- q ~ 1
C1<-fitcopula(Q, loc, "gaussian","whitmat", loc.form, scale.form,
shape.form, method = "Nelder")
print(C1)
loc.form <- q ~ lat
scale.form <- q ~ lat+lon
shape.form <- q ~ 1
C2<-fitcopula(Q, loc, "gaussian","whitmat", loc.form, scale.form,
shape.form, method = "Nelder")
print(C2)
loc.form <- q ~ lon
scale.form <- q ~ lat
shape.form <- q ~ 1
C4<-fitcopula(Q, loc, "gaussian","whitmat", loc.form, scale.form,
shape.form, method = "Nelder")
print(C3)
loc.form <- q ~ lon
scale.form <- q ~ lon
shape.form <- q ~ 1
C5<-fitcopula(Q, loc, "gaussian","whitmat", loc.form, scale.form,
shape.form, method = "Nelder")
print(C4)
loc.form <- q ~ lon
scale.form <- q ~ lat+lon
```

```
shape.form <- q ~ 1
C6<-fitcopula(Q, loc, "gaussian","whitmat", loc.form, scale.form,
shape.form, method = "Nelder")
print(C5)
loc.form <- q ~ lat+lon
scale.form <- q ~ lat+lon
shape.form <- q ~ 1
C9<-fitcopula(Q, loc, "gaussian","whitmat", loc.form, scale.form,
shape.form, method = "Nelder")
print(C6)
loc.form <- q ~ lon
scale.form <- q ~ lon+lat
shape.form <- q ~ 1
C10<-fitcopula(Q, loc, "gaussian","whitmat", loc.form, scale.form,
shape.form, method = "Nelder")
print(C7)
loc.form <- q ~ lat+lon
scale.form <- q ~ lon+lat
shape.form <- q ~ 1
C12<-fitcopula(Q, loc, "gaussian","whitmat", loc.form, scale.form,
shape.form, method = "Nelder")
print(C8)
#return level
predict(C1, loc, ret.per=20)
#transformasi copula ke gev
c1 <- log (0.95711)
c2 <- log (0.971285)
c3 <- log (0.9487879)
c4 <- log (0.990686)
c5 <- log (0.986658)
c6 <- log (0.970127)
c7 <- log (0.977749)
c8 <- log (0.939114)
c9 <- log (0.957274)
c10 <- log (0.957318)
c11 <- log (0.964963)
f1 <- -1/c1
f2 <- -1/c2
f3 <- -1/c3
f4 <- -1/c4
f5 <- -1/c5
f6 <- -1/c6
f7 <- -1/c7
f8 <- -1/c8
f9 <- -1/c9
f10 <- -1/c10
f11<- -1/c11
a1 <- frech2gev(f1, 55.28092,39.33090,-0.30452)
a2 <- frech2gev(f2, 60.87602,41.88608,-0.53381)
a3 <- frech2gev(f3, 55.73079,39.66276,-0.19016)
a4 <- frech2gev(f4, 39.40173,32.24765,-0.19991)
a5 <- frech2gev(f5, 44.98968,30.36494,-0.09754)
a6 <- frech2gev(f6, 62.06489,39.86060,-0.34936)
a7 <- frech2gev(f7, 50.41075,36.59654,-0.30406)
a8 <- frech2gev(f8, 53.98772,35.69147,-0.07981)
a9 <- frech2gev(f9, 59.54021,37.57842,-0.27829)
a10 <- frech2gev(f10, 57.9003,39.52390,-0.21433)
a11 <- frech2gev(f11, 53.56826,38.54955,-0.27580)
```

Lampiran 11. *Output* Parameter GEV

```
GEV fit
-----------------------------------
Response variable: gemarang


 L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  55.98519
Scale (sigma):  40.85714
Shape (xi):  -0.3498116

 Likelihood ratio test (5% level) for xi=0 does not accept Gumbel
hypothesis.
likelihood ratio statistic is  10.456  >  3.841459  1 df chi-
square critical value.

 p-value for likelihood-ratio test is  0.001222516


 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    55.28092     4.86927
SIGMA: (identity) 39.33090     3.48053
Xi: (identity)    -0.30452     0.07050

   GEV fit
-----------------------------------
Response variable: guyung


 L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  59.96493
Scale (sigma):  42.70229
Shape (xi):  -0.4993454

 Likelihood ratio test (5% level) for xi=0 does not accept Gumbel
hypothesis.
likelihood ratio statistic is  26.21485  >  3.841459  1 df chi-
square critical value.

 p-value for likelihood-ratio test is  3.054638e-07


 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    60.87602     5.06728
SIGMA: (identity) 41.88608     3.96833
Xi: (identity)    -0.53381     0.06733
```

```
   GEV fit
-----------------------------------
Response variable: karangjati
```

```
 L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  56.42036
Scale (sigma):  39.72231
Shape (xi):  -0.2212119

 Likelihood ratio test (5% level) for xi=0 does not accept Gumbel
hypothesis.
likelihood ratio statistic is  6.058177  >  3.841459  1 df chi-
square critical value.

 p-value for likelihood-ratio test is  0.01384205


 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    55.73079     4.87788
SIGMA: (identity) 39.66267     3.36032
Xi: (identity)    -0.19016     0.06049

  GEV fit
-----------------------------------
Response variable: kedungbendo


 L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  38.64783
Scale (sigma):  33.24491
Shape (xi):  -0.1718326

 Likelihood ratio test (5% level) for xi=0 does not reject Gumbel
hypothesis.
likelihood ratio statistic is  2.77924  <  3.841459  1 df chi-
square critical value.

 p-value for likelihood-ratio test is  0.09549347


 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    39.40173     4.24255
SIGMA: (identity) 32.24765     3.16806
Xi: (identity)    -0.19991     0.10883

   GEV fit
-----------------------------------
Response variable: kedunggalar
```

```
 L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  45.71288
Scale (sigma):  31.7098
Shape (xi):  -0.1528849

 Likelihood ratio test (5% level) for xi=0 does not reject Gumbel
hypothesis.
likelihood ratio statistic is  1.398921  <  3.841459  1 df chi-
square critical value.

 p-value for likelihood-ratio test is  0.2369043


 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                        MLE Stand. Err.
MU: (identity)    44.99985     3.80173
SIGMA: (identity) 30.34937     2.69522
Xi: (identity)    -0.09754     0.07318

   GEV fit
-----------------------------------
Response variable: kendal


 L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  63.43404
Scale (sigma):  41.32941
Shape (xi):  -0.4259181

 Likelihood ratio test (5% level) for xi=0 does not accept Gumbel
hypothesis.
likelihood ratio statistic is  15.08382  >  3.841459  1 df chi-
square critical value.

 p-value for likelihood-ratio test is  0.0001028409


 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    62.06489     4.90201
SIGMA: (identity) 39.86060     3.49048
Xi: (identity)    -0.34936     0.06665

   GEV fit
-----------------------------------
Response variable: kricak



 L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  51.52375
Scale (sigma):  38.34578
Shape (xi):  -0.3748595
```

```
 Likelihood ratio test (5% level) for xi=0 does not accept Gumbel
hypothesis.
likelihood ratio statistic is  10.95357  >  3.841459  1 df chi-
square critical value.

 p-value for likelihood-ratio test is  0.0009342333


 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    50.41075     4.50407
SIGMA: (identity) 36.59654     3.20674
Xi: (identity)    -0.30406     0.06539

   GEV fit
-----------------------------------
Response variable: mantingan


 L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  54.99635
Scale (sigma):  35.71235
Shape (xi):  -0.127921

 Likelihood ratio test (5% level) for xi=0 does not reject Gumbel
hypothesis.
likelihood ratio statistic is  1.410911  <  3.841459  1 df chi-
square critical value.

 p-value for likelihood-ratio test is  0.2349053


 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    53.98772     4.39634
SIGMA: (identity) 35.69147     3.03460
Xi: (identity)    -0.07981     0.05920

   GEV fit
-----------------------------------
Response variable: mardisari


 L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  59.6804
Scale (sigma):  38.44175
Shape (xi):  -0.295153

 Likelihood ratio test (5% level) for xi=0 does not accept Gumbel
hypothesis.
likelihood ratio statistic is  9.730872  >  3.841459  1 df chi-
square critical value.
```

```
 p-value for likelihood-ratio test is  0.001811985

 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    59.54021     4.66253
SIGMA: (identity) 37.57842     3.29405
Xi: (identity)    -0.27829     0.07160

   GEV fit
-----------------------------------
Response variable: papungan

 L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  55.06195
Scale (sigma):  38.94324
Shape (xi):  -0.3099866

 Likelihood ratio test (5% level) for xi=0 does not accept Gumbel
hypothesis.
likelihood ratio statistic is  4.550871  >  3.841459  1 df chi-
square critical value.

 p-value for likelihood-ratio test is  0.03290201

 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    57.90030     4.75371
SIGMA: (identity) 39.52390     3.45866
Xi: (identity)    -0.21433     0.05963

  GEV fit
-----------------------------------
Response variable: paron

  L-moments (stationary case) estimates (used to initialize MLE
optimization routine):
Location (mu):  53.38647
Scale (sigma):  39.59441
Shape (xi):  -0.2783413

 Likelihood ratio test (5% level) for xi=0 does not accept Gumbel
hypothesis.
likelihood ratio statistic is  8.299162  >  3.841459  1 df chi-
square critical value.

 p-value for likelihood-ratio test is  0.003966338


 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    53.56826     4.80590
SIGMA: (identity) 38.54955     3.44102
Xi: (identity)    -0.27580     0.07527
```

Lampiran 12. Output R Estimasi Parameter Copula

```
print(C1)
          Copula: gaussian
        Deviance: 8340.333
             AIC: 8356.333
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
  -455.09     -68.06
       Scale Parameters:
scaleCoeff1  scaleCoeff2
   135.5712      -0.8846
       Shape Parameters:
shapeCoeff1
    -0.1578
  Dependence Parameters:
 nugget    range   smooth
0.02645  2.78951  0.12998

Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
    0.15982      2.20193      0.05588    135.83059     18.24974
scaleCoeff1 scaleCoeff2  shapeCoeff1
  873.71463  7.84385      0.02275

 print(C2)
          Copula: gaussian
        Deviance: 8342.068
             AIC: 8360.068
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
   -458.1      -68.5
       Scale Parameters:
scaleCoeff1  scaleCoeff2  scaleCoeff3
    116.618       12.290        0.106
       Shape Parameters:
shapeCoeff1
      -0.16
  Dependence Parameters:
 nugget    range   smooth
0.03677  2.19733  0.14093

Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
    0.14848      1.40134      0.05564    134.63463     18.06237
scaleCoeff1    scaleCoeff2  scaleCoeff3  shapeCoeff1
   13.90570      5.60146      0.02277     612.85427
```

```
print(C3)
           Copula: gaussian
         Deviance: 8399.206
              AIC: 8415.206
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
  1317.07     -11.36
       Scale Parameters:
scaleCoeff1  scaleCoeff2
     133.18        13.05
       Shape Parameters:
shapeCoeff1
    -0.1407
  Dependence Parameters:
 nugget    range   smooth
 0.4700  12.0095   0.4354

Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
     0.04194           NA           NA    688.80300      6.18347
scaleCoeff1  scaleCoeff2  shapeCoeff1
   99.61339  13.36758      0.02318

print(C4)
           Copula: gaussian
         Deviance: 8365.033
              AIC: 8381.033
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
  1391.07     -12.03
       Scale Parameters:
scaleCoeff1  scaleCoeff2
    162.434       -1.132
       Shape Parameters:
shapeCoeff1
    -0.1618
  Dependence Parameters:
nugget   range  smooth
0.3123  0.6825  0.4086

Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
   1.194e-01    6.117e-01    3.295e-01    2.122e+03    1.905e+01
scaleCoeff1  scaleCoeff2  shapeCoeff1
  7.006e+02   6.290e+00    2.218e-02

  print(C4)
           Copula: gaussian
         Deviance: 8365.033
              AIC: 8381.033
```

```
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
  1391.07     -12.03
      Scale Parameters:
scaleCoeff1  scaleCoeff2
    162.434       -1.132
      Shape Parameters:
shapeCoeff1
    -0.1618
  Dependence Parameters:
nugget   range  smooth
0.3123  0.6825  0.4086

Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
   1.194e-01    6.117e-01    3.295e-01    2.122e+03    1.905e+01
scaleCoeff1  scaleCoeff2  shapeCoeff1
  7.006e+02  6.290e+00    2.218e-02

C5
             Copula: gaussian
         Deviance: 8369.024
              AIC: 8387.024
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
  1238.42     -10.66
      Scale Parameters:
scaleCoeff1  scaleCoeff2  scaleCoeff3
   1537.422       -5.496      -13.836
      Shape Parameters:
shapeCoeff1
    -0.1582
  Dependence Parameters:
nugget   range  smooth
0.2956  0.5642  0.4663

Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
       NA           NA           NA    1.353e+03    1.215e+01
 scaleCoeff1  scaleCoeff2  scaleCoeff3  shapeCoeff1
  8.236e+02  1.601e+01    7.466e+00    2.324e-02

   print(C6)
           Copula: gaussian
         Deviance: 8341.167
              AIC: 8361.167
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
```

```
    Location Parameters:
locCoeff1  locCoeff2  locCoeff3
  1087.63     -65.91     -13.71
       Scale Parameters:
scaleCoeff1  scaleCoeff2  scaleCoeff3
   150.8261       8.2632      -0.4715
       Shape Parameters:
shapeCoeff1
     -0.157
  Dependence Parameters:
nugget   range  smooth
0.1247  2.0265  0.1712

Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
   1.070e-01    1.329e+00    6.044e-02    1.899e+03    1.852e+01
locCoeff3  scaleCoeff1  scaleCoeff2  scaleCoeff3  shapeCoeff1
 1.675e+01   1.241e+03    1.400e+01    1.119e+01    2.309e-02


print(C7)
           Copula: gaussian
         Deviance: 8341.167
              AIC: 8361.167
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2  locCoeff3
  1087.63     -65.91     -13.71
       Scale Parameters:
scaleCoeff1  scaleCoeff2  scaleCoeff3
   150.8261      -0.4715       8.2632
       Shape Parameters:
shapeCoeff1
     -0.157
  Dependence Parameters:
nugget   range  smooth
0.1247  2.0265  0.1712

Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
    0.10683      1.32880      0.06044    870.21868     17.91402
locCoeff3  scaleCoeff1  scaleCoeff2  scaleCoeff3  shapeCoeff1
  7.77197  817.72668      7.42236     14.00584      0.02292


print(C8)
           Copula: gaussian
         Deviance: 8369.024
              AIC: 8387.024
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
```

```
  1238.42     -10.66
       Scale Parameters:
scaleCoeff1  scaleCoeff2  scaleCoeff3
   1537.422      -13.836       -5.496
       Shape Parameters:
shapeCoeff1
    -0.1582
  Dependence Parameters:
nugget   range  smooth
0.2956  0.5642  0.4663

Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
         NA           NA           NA    761.54975      6.83499
scaleCoeff1  scaleCoeff2  scaleCoeff3  shapeCoeff1
  667.21748    6.08924     16.00976      0.02288
print(C9)
           Copula: gaussian
         Deviance: 8342.916
              AIC: 8358.916
Covariance Family: Whittle-Matern
Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
   -433.5      -65.1
       Scale Parameters:
scaleCoeff1  scaleCoeff2
     59.602        2.902
       Shape Parameters:
shapeCoeff1
    -0.1468
  Dependence Parameters:
   nugget      range     smooth
3.928e-07  1.051e+01  9.804e-02

print(C10)
           Copula: gaussian
         Deviance: 8340.281
              AIC: 8358.281
Covariance Family: Whittle-Matern
Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2  locCoeff3
  1109.67     -92.84     -15.70
       Scale Parameters:
scaleCoeff1  scaleCoeff2
     66.476        3.956
       Shape Parameters:
shapeCoeff1
    -0.1594
  Dependence Parameters:
   nugget      range     smooth
8.487e-06  2.660e+00  1.258e-01
Optimization Information
  Convergence: successful
  Function Evaluations: 2506
```

```
print(C11)
           Copula: gaussian
         Deviance: 8338.167
              AIC: 8365.167
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2  locCoeff3
  1110.37     -76.11     -14.59
       Scale Parameters:
scaleCoeff1  scaleCoeff2
   133.3973      -0.8658
       Shape Parameters:
shapeCoeff1
     -0.159
  Dependence Parameters:
   nugget      range     smooth
1.229e-06  3.039e+00  1.209e-01

print(C12)
           Copula: gaussian
         Deviance: 8342.068
              AIC: 8360.068
Covariance Family: Whittle-Matern
Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
   -458.1      -68.5
       Scale Parameters:
scaleCoeff1  scaleCoeff2  scaleCoeff3
    116.618        0.106       12.290
       Shape Parameters:
shapeCoeff1
      -0.16
  Dependence Parameters:
 nugget    range   smooth
0.03677  2.19733  0.14093
Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
    0.14850      1.40135      0.05564    134.44342     18.03866
scaleCoeff1 scaleCoeff2  scaleCoeff3  shapeCoeff1
    753.64588  6.85214     13.90629      0.02277

print(C13)
           Copula: gaussian
         Deviance: 8340.281
              AIC: 8358.281
Covariance Family: Whittle-Matern
Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2  locCoeff3
  1109.67     -15.70     -92.84
       Scale Parameters:
```

```
scaleCoeff1  scaleCoeff2
     66.476        3.956
       Shape Parameters:
shapeCoeff1
    -0.1594
  Dependence Parameters:
   nugget      range     smooth
8.487e-06  2.660e+00  1.258e-01
Optimization Information
  Convergence: successful
  Function Evaluations: 2506

print(C14)
           Copula: gaussian
         Deviance: 8338.167
              AIC: 8365.167
Covariance Family: Whittle-Matern
Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2  locCoeff3
  1110.37     -14.59     -76.11
       Scale Parameters:
scaleCoeff1  scaleCoeff2
   133.3973      -0.8658
       Shape Parameters:
shapeCoeff1
     -0.159
  Dependence Parameters:
   nugget      range     smooth
1.229e-06  3.039e+00  1.209e-01
Optimization Information
  Convergence: successful
  Function Evaluations: 2284
print(C15)
           Copula: gaussian
         Deviance: 8342.068
              AIC: 8360.068
Covariance Family: Whittle-Matern
Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
   -458.1      -68.5
       Scale Parameters:
scaleCoeff1  scaleCoeff2  scaleCoeff3
    116.618        0.106       12.290
       Shape Parameters:
shapeCoeff1
      -0.16
  Dependence Parameters:
 nugget    range   smooth
0.03677  2.19733  0.14093
Standard Errors
     nugget        range       smooth    locCoeff1    locCoeff2
    0.14850      1.40135      0.05564    134.44342     18.03866
scaleCoeff1  scaleCoeff2  scaleCoeff3  shapeCoeff1
 753.64588     6.85214     13.90629      0.02277
```

Lampiran 13. Turunan Kedua Fungsi *Ln Pairrwise Likelihood* Copula Gaussian terhadap parameter $\boldsymbol{\beta}_{\boldsymbol{\mu}}$

$$
\frac{\partial^2 l(\boldsymbol{\beta})}{\partial\boldsymbol{\beta}_{\boldsymbol{\mu}}\partial\boldsymbol{\beta}_{\boldsymbol{\mu}}}=
\frac{\partial\left(\begin{aligned}
&\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\Bigg(\Bigg(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\\
&\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\Bigg)\Bigg).\\
&\Bigg[\frac{1}{2}\left(\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{T}.\\
&\left(\rho(h)^{-1}\right)\cdot\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\Bigg]\ \text{-0.5}\ln|\rho(h)|
\end{aligned}\right)}{\partial\boldsymbol{\beta}_{\boldsymbol{\mu}}\partial\boldsymbol{\beta}_{\boldsymbol{\mu}}}
$$

$$A = \ln\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right.$$

$$\left.\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)$$

$$\frac{\partial A}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right) +\right.$$

$$\left.\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right] +$$

$$
\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\cdot\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{\mathbf{k}})}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right.
$$

$$
\left.+\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{\mathbf{k}})}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}.\right)\right]
$$

Misalkan $\frac{\partial A}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = (x+y)+(w+z)$

maka $\frac{\partial A}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \frac{\partial (x+y)}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} + \frac{\partial (w+z)}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}}$

$$
\frac{\partial (x+y)}{\partial \boldsymbol{\beta}_{\mu}} = \left(\frac{\dfrac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\dfrac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}-1}}{\left(\dfrac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\dfrac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right)^{2}} \cdot \left\{ \frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\left(-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}-1} \right\} + \right.
$$

$$
\left. \frac{1}{\left\{\dfrac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\dfrac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}} \cdot \frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\left(1-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}-1} \cdot \frac{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{j}\right)}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}} \cdot \frac{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{j}\right)}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}} \right)
$$

$$
\frac{\partial (w+z)}{\partial \boldsymbol{\beta}_{\mu}} = \left(\frac{\dfrac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\dfrac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}}{\left(\dfrac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\dfrac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right)^{2}} \cdot \left\{ \frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1} \right\} +
$$

$$
\left. \frac{1}{\left\{\dfrac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\dfrac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}} \cdot \frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1} \cdot \frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \cdot \frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right)
$$

Sehingga

$$
\frac{\partial A}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \frac{\partial (x+y)}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} + \frac{\partial (w+z)}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}}
$$

$$
\frac{\partial A}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}} \partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \left(\frac{\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \left[1 + \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi}} - 1}}{\left(\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi}}} \right)^2} \cdot \left\{ \frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left(-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \right) \left[1 + \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi}} - 1} \right\} + \right.
$$

$$
\left. \frac{1}{\left\{ \frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi}}} \right\}} \cdot \frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left(1 - \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \right) \left[1 + \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1 - \mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi}} - 1} \cdot \frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \cdot \left(-\mathbf{d}_{\mathrm{j}} \right)}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \cdot \frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \cdot \left(-\mathbf{d}_{\mathrm{j}} \right)}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) +
$$

$$
\left(\frac{\frac{1}{\mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \left[1 + \mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\xi}} - 1}}{\left(\frac{1}{\mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\xi}}} \right)^2} \cdot \left\{ \frac{1}{\mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left(-\mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \right) \left[1 + \mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{k}^{\mathrm{T}} \boldsymbol{\beta}_{\xi}} - 1} \right\} + \right.
$$

$$
\frac{1}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_k\right)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\cdot\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_k\right)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)
$$

$$
B=\left[\begin{array}{l}
\frac{1}{2}\left[\left[\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]\right]\left[\Phi^{-1}\left[\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right]\right]\\
\left(\rho\left(h\right)^{-1}\right)\cdot\left[\left[\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right)\right]\right]\left[\left[\Phi^{-1}\left[\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right]\right]\right]^{T}
\end{array}\right]
$$

$$
\begin{aligned}
\frac{\partial B}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \frac{1}{2} \Bigg[\Bigg[& \phi^{-1}\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) . \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} . \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} . \left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(- \mathbf{d}_{\mathbf{j}} \right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \Bigg] \\
& \Bigg[\phi^{-1}\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T} \beta_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) . \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} . \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} . \left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(- \mathbf{d}_{\mathbf{k}} \right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \Bigg] \\
& . \left(\rho \left(h \right)^{-1} \right) . \left(\Phi^{-1}\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \Phi^{-1}\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{T} + \\
& \Bigg[\Bigg[\phi^{-1}\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) . \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} . \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} . \left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(- \mathbf{d}_{\mathbf{j}} \right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \Bigg] \\
& \Bigg[\phi^{-1}\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T} \beta_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) . \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} . \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} . \left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(- \mathbf{d}_{\mathbf{k}} \right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \Bigg]^{T} \\
& . \left(\rho \left(h \right)^{-1} \right) . \left(\Phi^{-1}\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \Phi^{-1}\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)
\end{aligned}
$$

$$\frac{\partial B}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} = \frac{1}{2}\left(a_1 \cdot a_2 \cdot a_3^{T} + a_1 \cdot^{T} a_2 \cdot a_3\right),\ \text{misalkan } a_1 \cdot a_2 = a_{11} \text{ maka}$$

$$\frac{\partial B}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} = \frac{1}{2}\left(\left(\frac{\partial a_{11}}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot a_3 + a_{11} \cdot \frac{\partial a_3}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}}\right) + \left(\frac{\partial a_{11}}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot a_3 + a_{11} \cdot \frac{\partial a_3}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}}\right)\right)$$

$$\frac{\partial a_{11}}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} = \frac{\partial a_1}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot a_2 + a_1 \cdot \frac{a_2}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}}, \frac{a_2}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} = 0,\ \text{maka } \frac{\partial a_{11}}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} = \frac{\partial a_1}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot a_2$$

$$a_1 = \left(u_1, u_2\right)$$

$$\frac{\partial a_1}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} = \left(\frac{\partial u_1}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}}, \frac{\partial u_2}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}}\right)$$

$$u_1 = a \cdot b \cdot c \cdot d\ \text{,misalkan } a \cdot b = k \text{ dan } c.d = l.$$

$$\frac{\partial u_1}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} = \frac{\partial k}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} l + \frac{\partial l}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} k$$

$$= \left(\left(\frac{\partial a}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot b + \frac{\partial b}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot a\right) c \cdot d + \left(\frac{\partial c}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot d + \frac{\partial d}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot c\right) a \cdot b\right), \frac{\partial d}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} = 0 \text{ maka } \left(\frac{\partial a}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot b + \frac{\partial b}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot a\right) c \cdot d + \left(\frac{\partial c}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} \cdot d\right) a \cdot b$$

$$a = \left[\phi^{-1}\left(\exp\left\{-\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}}\right\}\right)\right.$$

$$\frac{\partial a}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}} \partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{-\frac{3}{4}} \cdot \left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{2}\right) \cdot \frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}}{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\right)^{3}} \cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right) + \left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}}\right)^{\frac{1}{2}} \cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right) \cdot \exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\} \cdot \frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}.$$

$$\mathrm{b} = \exp\left\{ -\left[1 + \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}$$

$$\frac{\partial b}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} = \exp\left\{ -\left[1 + \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \cdot \frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \cdot \left(\frac{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(-\mathbf{d}_{\mathrm{j}} \right)}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right)$$

$$c = \frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1}$$

$$\frac{\partial c}{\partial \boldsymbol{\beta}_{\mu} \partial \boldsymbol{\beta}_{\mu}} = \frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left(-\frac{1 - \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \right) \left[1 + \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1 - \mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \left(\frac{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(-\mathbf{d}_{\mathrm{j}} \right)}{\mathbf{d}_{\mathrm{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right)$$

$$\frac{\partial u_1}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}} \partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \left(\left(\frac{\partial a}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}} \partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} \cdot b + \frac{\partial b}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}} \partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} \cdot a \right) c \cdot d + \left(\frac{\partial c}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}} \partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} \cdot d \right) a \cdot b \right)$$

$$= \frac{ -\frac{1}{4} \left(2\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{-\frac{3}{4}} \cdot \left(6\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right)^{2} \cdot \frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \cdot \left(-\mathbf{d}_{\mathbf{j}} \right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) }{ \left(2\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{3} } \cdot$$

$$\exp\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) + \left(\frac{1}{\sqrt{ \left(2\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{3} }} \right)^{\frac{1}{2}}$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi.\left(-\mathbf{d}_j\right)}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}+\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot\left(\frac{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi.\left(-\mathbf{d}_j\right)}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\cdot$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\cdot\left(\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\right)\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot$$

$$\left.\left(\frac{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi.\left(-\mathbf{d}_j\right)}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)\cdot\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}$$

$$\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\left(\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left(-\frac{1-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\right)\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\right)\cdot\left(\frac{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi.\left(-\mathbf{d}_j\right)}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)$$

Dengan cara permisalan yang sama maka penurunan u_2 adalah

$$\frac{\partial u_2}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}} \partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{-\frac{3}{4}} \cdot \left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{2}\right) \cdot \frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}} \cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right) + \left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}}\right)^{\frac{1}{2}}$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\cdot$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\cdot\left(\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\right)\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\right.\cdot$$

$$\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}$$

$$\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\left(\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\right)\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)$$

$$
\begin{aligned}
\frac{\partial a_3}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}} \partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \Bigg[\Bigg[& \phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) . \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} . \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} . \left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(-\mathbf{d}_{\mathbf{j}} \right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \Bigg] \\
& \Bigg[\phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_k^T \beta_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) . \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} . \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} . \left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \left(-\mathbf{d}_{\mathbf{k}} \right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \Bigg]
\end{aligned}
$$

$$\frac{\partial B}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}} \partial \boldsymbol{\beta}_{\boldsymbol{\mu}}} = \frac{1}{2}\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{-\frac{3}{4}} \cdot \left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{2}\right) \cdot \frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \cdot \left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}} \cdot \right.\right.$$

$$\left.\left. \exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right) \cdot + \left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{3}}}\right)^{\frac{1}{2}}\right.\right.$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathrm{j}}\right)}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(\frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathrm{j}}\right)}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\cdot$$

$$+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\cdot\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\left(\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\right)\cdot\left(\frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathrm{j}}\right)}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\cdot$$

$$
\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}.x_{ki}\right)^2\right)\cdot\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.\left(-\mathbf{d}_k\right)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}}{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}.\right)\right)^3}.
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}.\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)^3}}\right)^{\frac{1}{2}}
$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\cdot$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\cdot\left(\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\cdot$$

$$\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\left(\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg)^{T}\cdot\left(\rho(h)^{-1}\right)\cdot$$

$$\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}$$

$$\cdot\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{\mathbf{j}})}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]\Bigg[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_k^T\beta_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\quad\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}$$

$$\cdot\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{\mathbf{k}})}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\Bigg]\cdot\left(\rho(h)^{-1}\right).\left\{\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}$$

$$\cdot\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{\mathbf{j}})}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\Bigg]\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\quad\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}$$

$$\cdot\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{\mathbf{k}})}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\Bigg]$$

$$\left(\frac{ -\frac{1}{4}\left(2\pi \left(\exp\left\{ -\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{-\frac{3}{4}} \cdot \left(6\pi \left(\exp\left\{ -\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \right)^{2} \right) \cdot \frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} }{ \left(2\pi \left(\exp\left\{ -\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{3} } \cdot \right.$$

$$\exp\left(\exp\left\{ -\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \right) + \left(\frac{1}{\sqrt{ \left(2\pi \left(\exp\left\{ -\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{3} }} \right)^{\frac{1}{2}}$$

$$\exp\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}.x_{ji}\Bigg)\cdot\exp\Bigg\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\cdot\frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\cdot(-\mathbf{d}_{\mathrm{j}})}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\cdot$$

$$\exp\Bigg\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}+\exp\Bigg\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\cdot\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(\frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\cdot(-\mathbf{d}_{\mathrm{j}})}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\cdot$$

$$\Bigg(\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg)\Bigg)\Bigg)\cdot\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg)\Bigg(\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}-1}\Bigg)\Bigg)\cdot\left(\frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\cdot(-\mathbf{d}_{\mathrm{j}})}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\cdot$$

$$-\frac{\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{2}\right)\cdot\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}.$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}}\right)^{\frac{1}{2}}$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\cdot\left(-\mathbf{d}_k\right)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\cdot\left(-\mathbf{d}_k\right)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\cdot$$

$$\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)\cdot\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\left(\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\right)\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\cdot\left(-\mathbf{d}_k\right)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)$$

$$\frac{\partial C}{\partial \boldsymbol{\beta}_\mu}=0$$

$$\frac{\partial C}{\partial \boldsymbol{\beta}_\mu \partial \boldsymbol{\beta}_\mu}=0$$

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_\mu \partial \boldsymbol{\beta}_\mu} = \frac{\partial (A \cdot B - C)}{\partial \boldsymbol{\beta}_\mu \partial \boldsymbol{\beta}_\mu}$$

$$= \left(\frac{\partial A}{\partial \boldsymbol{\beta}_\mu \partial \boldsymbol{\beta}_\mu} B + A \frac{\partial B}{\partial \boldsymbol{\beta}_\mu \partial \boldsymbol{\beta}_\mu} \right)$$

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_\mu \partial \boldsymbol{\beta}_\mu} = 0$$

$$0 = \sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \left\{ \frac{\frac{1}{\mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\sigma} \mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\xi \left[1 + \mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\xi} - 1}}{\left(\frac{1}{\mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\sigma} \left[1 + \mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\xi}} \right)^2} \cdot \left\{ \frac{1}{\mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\sigma} \left(-\mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\xi \right) \left[1 + \mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}} \boldsymbol{\beta}_\xi} - 1} \right\} + \right.$$

$$\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\cdot\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(1-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\cdot\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)+$$

$$\Bigg[\Bigg(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\cdot\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{k}}\right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg)$$

$$+\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{\mathbf{k}})}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]+$$

$$\left(\frac{\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}}{\left(\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right)^{2}}\cdot\left\{\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}+$$

$$\frac{1}{\left\{\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\cdot\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(1-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\frac{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot(-\mathbf{d}_{k})}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\cdot\frac{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{k})}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)$$

$$
\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)+\right.
$$

$$
\left.\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right].
$$

$$
\begin{aligned}
&\frac{1}{2}\cdot\Bigg[\Bigg[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]\\
&\Bigg[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{k}}\right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]^{T}\\
&\cdot\left(\rho\left(h\right)^{-1}\right).\Bigg(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\\
&\cdot\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]\Bigg[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\\
&\cdot\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{k}}\right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]+
\end{aligned}
$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)+\right.$$

$$\left.\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(\mathbf{-d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right].$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\cdot\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{\mathbf{k}})}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right.$$

$$\left.+\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_{\mathbf{k}})}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right].$$

$$
\frac{1}{2}\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{2}\right)\cdot\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}}{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{3}}\right.\right.\cdot
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{3}}}\right)^{\frac{1}{2}}
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_j\right)}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\cdot
$$

$$
\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(\frac{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_j\right)}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\cdot
$$

$$
\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\cdot\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\left(\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(\frac{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_j\right)}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\cdot
$$

$$
\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{2}\right)\cdot\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}}{2\pi\left(e\,xp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}.
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}}\right)^{\frac{1}{2}}
$$

$$\exp\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}.x_{ki}\Bigg)\cdot\exp\Bigg\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\cdot$$

$$\exp\Bigg\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}+\exp\Bigg\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\cdot$$

$$\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg)\Bigg)\cdot\Bigg(\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\Bigg)+\Bigg(\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg)\Bigg)\cdot$$

$$\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg)\Bigg(\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\Bigg)\Bigg)\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\cdot x_{ki}\Bigg)^{T}\cdot\left(\rho(h)^{-1}\right)\cdot$$

$$\Bigg(\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg)\Bigg)\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}.\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}$$

$$\cdot\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\xi.(-\mathbf{d}_{\mathbf{j}})}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\Bigg]\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\mu}{d_k^{T}\beta_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg).\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}.\quad\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}$$

$$
\cdot\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{k}}\right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]\cdot\left(\rho\left(h\right)^{-1}\right).\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}
$$

$$
\cdot\left(\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{j}}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg],\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\quad\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}
$$

$$
\cdot\left(\frac{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{\mathbf{k}}\right)}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]
$$

$$\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{2}\right)\cdot\frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathrm{j}}\right)}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}\cdot \right.$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}}\right)^{\frac{1}{2}}$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathrm{j}}\right)}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(\frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathrm{j}}\right)}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\cdot$$

$$\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\cdot\left(\exp\left\{-\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\left(\frac{1}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(\frac{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathrm{j}}\right)}{\mathbf{d}_{\mathrm{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\cdot$$

$$
\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{2}\right)\cdot\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_k\right)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}.
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}}\right)^{\frac{1}{2}}
$$

$$
e\,\mathrm{xp}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)
$$

$$
\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right).
$$

$$
\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\cdot\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\left(\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.(-\mathbf{d}_k)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)
$$

Lampiran 14. Turunan Kedua Fungsi *Ln Pairrwise Likelihood* Copula Gaussian terhadap parameter $\boldsymbol{\beta}_{\boldsymbol{\sigma}}$

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \frac{\partial \left(\begin{array}{l} \displaystyle\sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \ln \left(\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}. \right. \\ \left. \left\{ \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right). \\ \left[\frac{1}{2} \left(\Phi^{-1} \left(\exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \Phi^{-1} \left(\exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{T} . \right. \\ \left. \left(\rho(h)^{-1} \right) \cdot \left(\Phi^{-1} \left(\exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \Phi^{-1} \left(\exp \left\{ - \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right) \right] \text{-}0.5 \ln \left| \rho(h) \right| \end{array} \right)}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}}$$

$$\frac{\partial^2 l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \frac{\partial (A \cdot B - C)}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}}, \frac{\partial (C)}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = 0 \text{ maka}$$

$$= \frac{\partial A}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} B + \frac{\partial B}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \mathrm{A}$$

$$A=\ln\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right.$$

$$\left.\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)$$

$$\frac{\partial A}{\partial\boldsymbol{\beta}_{\boldsymbol{\sigma}}}=\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}+\right.\right.$$

$$\left.\left.\left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}+\frac{2\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}}\right)\right)+\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)\right]+$$

$$
\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}+\right.\right.
$$

$$
\left.\left.\left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}+\frac{2\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}}\right)\right)+\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\cdot\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)\right]
$$

Misalkan $\dfrac{\partial A}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}}=(x+y)\cdot(w+z)$

maka $\dfrac{\partial A}{\partial \boldsymbol{\beta}_{\sigma}\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}}=\dfrac{\partial (x+y)}{\partial \boldsymbol{\beta}_{\sigma}\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\dfrac{\partial (w+z)}{\partial \boldsymbol{\beta}_{\sigma}\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}}$

$$\frac{\partial(x+y)}{\partial\boldsymbol{\beta}_{\sigma}\partial\boldsymbol{\beta}_{\sigma}}=\Bigg(\frac{\left\{-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}-1}\right\}\cdot\left(\frac{\mathbf{d}_{j}}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)}{\left\{\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}^{2}}\left(\frac{\mathbf{d}_{j}}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)$$

$$\frac{1}{\left\{\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\right)^{2}\left[\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}-1}\right\}\left[\frac{\left(\mathbf{d}_{j}\right)}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right]$$

$$\left(\frac{\mathbf{d}_{j}}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)\Bigg)\Bigg)\Bigg)+\left(\frac{2\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)\mathbf{d}_{j}}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{4}}+\frac{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)\cdot\left(2\mathbf{d}_{j}\cdot\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\right).-2\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{j}^{T}\boldsymbol{\beta}\left(x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{8}}\right)$$

$$\Bigg(\frac{1}{\left\{\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}-1}\right\}.+$$

$$\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}-1\right)\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-2\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\frac{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_{\mathbf{j}}\right)^{2}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)+$$

$$\left(\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{2\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\left(\mathbf{d}_{\mathbf{j}}\right)^{2}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}}\right)\right).x_{ji}\right)\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}$$

$$\frac{\partial(w+z)}{\partial\boldsymbol{\beta}_{\boldsymbol{\sigma}}\partial\boldsymbol{\beta}_{\boldsymbol{\sigma}}}=\Bigg(\frac{\left\{-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\cdot\left(\frac{\mathbf{d}_k}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}+\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\right)}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}^2}\left(\frac{\mathbf{d}_k}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}+\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\right)$$

$$\frac{1}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)^2\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\left[\frac{\left(\mathbf{d}_k\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)^2}+\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\right]$$

$$\left(\frac{\left(\mathbf{d}_k\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)^2}+\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\right)\Bigg)\Bigg)\Bigg)+\left(\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\mathbf{d}_k}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}+\frac{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\left(2\mathbf{d}_k\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right).-2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^8}\right)$$

$$\Bigg(\frac{1}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.+$$

$$\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}-1\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_k\right)^2}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right).$$

$$\left(\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\left(\mathbf{d}_k\right)^2\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\right)\right).x_{ji}\right)\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}$$

$$\frac{\partial A}{\partial\boldsymbol{\beta}_o\partial\boldsymbol{\beta}_\sigma}=\frac{\partial(x+y)}{\partial\boldsymbol{\beta}_o\partial\boldsymbol{\beta}_\sigma}+\frac{\partial(w+z)}{\partial\boldsymbol{\beta}_o\partial\boldsymbol{\beta}_\sigma}$$

$$=\left(\frac{\left\{-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\cdot\left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}+\frac{2\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\right)}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}^2}\left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}+\frac{2\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\right)\right)$$

$$\frac{1}{\left\{\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}}\right\}}\left\{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\right)^{2}\left[\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^{2}}\right)\right]^{-\frac{1-\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}-1}\right\}\left[\frac{\left(\mathbf{d}_j\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^{2}}+\frac{2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)\cdot\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi.\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^{4}}\right]$$

$$\left(\frac{\mathbf{d}_j}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^{2}}+\frac{2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)\cdot\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi.\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^{4}}\right)\Bigg)\Bigg)+\left(\frac{2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)\mathbf{d}_j}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^{4}}+\frac{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)\cdot\left(2\mathbf{d}_j\cdot\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\right).-2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi.\mathbf{d}_j^{T}\boldsymbol{\beta}\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^{8}}\right)$$

$$\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}}\right\}}\left\{-\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left[\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}-1}\right\}+\right.$$

$$\left(\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\left(-\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi-1\right)\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1-2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\right)}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}-1}\cdot\frac{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi.\left(-\mathbf{d}_j\right)^{2}}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^{2}}\cdot\left(-\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^{2}}\right)\right)\right)+$$

$$\left(\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{2\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\left(\mathbf{d}_{\mathbf{j}}\right)^{2}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)\right).x_{ji}\right)\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\left.\vphantom{\frac{1}{1}}\right)$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\right.\right.$$

$$\left.\left.\left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)\right)\right.+$$

$$\left.\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)\right)\right]+$$

$$
\left(\frac{\left\{-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\cdot\left(\frac{\mathbf{d}_k}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}^{2}}\left(\frac{\mathbf{d}_k}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)\right.
$$

$$
\frac{1}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)^{2}\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\left[\frac{\left(\mathbf{d}_k\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)^{2}}+\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right]
$$

$$
\left.\left(\frac{\left(\mathbf{d}_k\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)^{2}}+\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)\right)\right)+\left(\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\mathbf{d}_k}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{4}}+\frac{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\cdot\left(2\mathbf{d}_k\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right).-2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{8}}\right)
$$

$$\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}+\right.$$

$$\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}-1\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_k\right)^2}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)+$$

$$\left.\left(\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\left(\mathbf{d}_k\right)^2\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\right)\right)\right)\right)\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\right.\right.$$

$$\left.\left.\left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}+\frac{2\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}}\right)\right)\right.+$$

$$\left.\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)\right]$$

$$
\frac{\partial b}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \frac{1}{2} \Bigg[\Bigg[\phi^{-1} \Bigg(\exp \Bigg\{ - \Bigg[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \bigg(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \bigg) \Bigg]^{- \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \Bigg\} \Bigg) . \exp \Bigg\{ - \Bigg[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \bigg(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \bigg) \Bigg]^{- \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \Bigg\} . \ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \Bigg[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \bigg(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \bigg) \Bigg]^{- \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1}
$$

$$
\cdot \Bigg(- \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \Bigg(\frac{- \mathbf{d}_{\mathbf{j}} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \Bigg) \Bigg) \Bigg] \Bigg] \ \Bigg[\phi^{-1} \Bigg(\exp \Bigg\{ - \Bigg[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \bigg(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T} \beta_{\sigma}} \bigg) \Bigg]^{- \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \Bigg\} \Bigg) . \ \exp \Bigg\{ - \Bigg[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \bigg(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \bigg) \Bigg]^{- \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \Bigg\} .
$$

$$
\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \Bigg[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \bigg(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \bigg) \Bigg]^{- \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} . \Bigg(- \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \Bigg(\frac{- \mathbf{d}_{\mathbf{k}} \left(x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \Bigg) \Bigg) \Bigg] \cdot \rho \left(h \right)^{-1} \cdot
$$

$$
\Bigg(\Phi^{-1} \Bigg(\exp \Bigg\{ - \Bigg[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \bigg(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \bigg) \Bigg]^{- \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \Bigg\} \Bigg) \ \Phi^{-1} \Bigg(\exp \Bigg\{ - \Bigg[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \bigg(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \bigg) \Bigg]^{- \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \Bigg\} \Bigg) \Bigg)^{T}
$$

$$\left(\phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) . \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} . \left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{j}^{T} \beta_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \right.$$

$$\left. \left. \cdot \left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{j}} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right) \right) \right) + \left[\phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \cdot \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right.$$

$$\left. \cdot \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \cdot \left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{k}} \left(x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right) \right]^{T} \cdot \rho\left(h \right)^{-1} .$$

$$\left(\Phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \Phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)$$

$$\frac{\partial B}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} = \frac{1}{2}\left(a_1 \cdot a_2 \cdot a_3{}^T + a_1 \cdot^T a_2 \cdot a_3\right), \text{ misalkan } a_1 \cdot a_2 = a_{11} \text{ maka}$$

$$\frac{\partial B}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} = \frac{1}{2}\left(\left(\frac{\partial a_{11}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot a_3 + a_{11} \cdot \frac{\partial a_3}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}}\right) + \left(\frac{\partial a_{11}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot a_3 + a_{11} \cdot \frac{\partial a_3}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}}\right)\right)$$

$$\frac{\partial a_{11}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} = \frac{\partial a_1}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot a_2 + a_1 \cdot \frac{a_2}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}}, \frac{a_2}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} = 0, \text{ maka } \frac{\partial a_{11}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} = \frac{\partial a_1}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot a_2$$

$$a_1 = \left(u_1, u_2\right)$$

$$\frac{\partial a_1}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} = \left(\frac{\partial u_1}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}}, \frac{\partial u_2}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}}\right)$$

$u_1 = a \cdot b \cdot c \cdot d$,misalkan $a \cdot b = k$ dan $c.d = l$.

$$\frac{\partial u_1}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} = \frac{\partial k}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} l + \frac{\partial l}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} k$$

$$= \left(\left(\frac{\partial a}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot b + \frac{\partial b}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot a\right) c \cdot d + \left(\frac{\partial c}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot d + \frac{\partial d}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot c\right) a \cdot b\right)$$

$$a = \left[\phi^{-1}\left(\exp\left\{-\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}}\right\}\right)\right.$$

$$
\frac{\partial a}{\partial \boldsymbol{\beta}_{\sigma}\partial \boldsymbol{\beta}_{\sigma}}=\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{2}\right)\cdot\left(-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{j}\left(x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{3}}\cdot
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{3}}}\right)^{\frac{1}{2}}\cdot
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}.\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{j}\left(x_{ji}-\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_{j}^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)
$$

$$b = \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}$$

$$\frac{\partial b}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \cdot \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \cdot \left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{j}} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right)$$

$$c = \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1}$$

$$\frac{\partial c}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left(-\frac{1 - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \right) \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1 - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \cdot \left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{j}} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right)$$

$$d = \left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{j}} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right)$$

$$\frac{\partial d}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \frac{2 \left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right) \cdot \left(\mathbf{d}_{j} \right)^{2} . \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{4}}$$

$$\frac{\partial u_{1}}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \frac{\partial k}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} l + \frac{\partial l}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} k$$

$$= \left(\left(\frac{\partial a}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \cdot b + \frac{\partial b}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \cdot a \right) c \cdot d + \left(\frac{\partial c}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \cdot d + \frac{\partial d}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \cdot c \right) a \cdot b \right)$$

$$= \left(\left(\left(\frac{ -\frac{1}{4} \left(2\pi \left(\exp\left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{-\frac{3}{4}} \cdot \left(6\pi \left(e\,xp\left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right)^{2} \right) \cdot \left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{j}} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right) }{ 2\pi \left(\exp\left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right)^{3} } \cdot \right. \right. \right.$$

$$\exp\left(\exp\left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) + \left(\frac{1}{\sqrt{2\pi \left(\exp\left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right)^{3}}} \right)^{\frac{1}{2}}$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot
$$

$$
\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\right)\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)
$$

$$
\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\right)
$$

$$
\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot x_{ji}\right)+\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\,\frac{2\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\cdot\left(\mathbf{d}_j\right)^2.\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^4}\right)\cdot\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)
$$

$$\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\Bigg)$$

Dengan cara permisalan yang sama maka penurunan u_2 adalah

$$
\frac{\partial u_2}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \left(\left(\left(\frac{ -\frac{1}{4}\left(2\pi \left(\exp\left\{ -\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} .x_{ki} \right) \right)^{-\frac{3}{4}} \cdot \left(6\pi \left(\exp\left\{ -\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} ..x_{ki} \right)^{2} \right) \cdot \left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right) }{ 2\pi \left(\exp\left\{ -\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} .x_{ki} \right)^{3} } \right.\right.\right. \cdot
$$

$$
\exp\left(\exp\left\{ -\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} .x_{ki} \right) + \left(\frac{1}{\sqrt{ 2\pi \left(\exp\left\{ -\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} .x_{ki} \right)^{3} }} \right)^{\frac{1}{2}}
$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot x_{ki}\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\right)\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)$$

$$\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\right)$$

$$\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot x_{ji}\right)+\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\,\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\cdot\left(\mathbf{d}_k\right)^2.\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^4}\cdot x_{ki}\cdot\phi^{-1}\left(\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)$$

$$\frac{\partial a_1}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \left(\frac{\partial u_1}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}}, \frac{\partial u_2}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right)$$

$$= \Bigg(\Bigg(\Bigg(\frac{ -\frac{1}{4} \left(2\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}. \right) \right)^{-\frac{3}{4}} \cdot \left(6\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right)^{2} \right) \cdot \left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{j}} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right) }{ 2\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}. \right)^{3} } .$$

$$\exp\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) + \left(\frac{1}{\sqrt{ 2\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right)^{3} }} \right)^{\frac{1}{2}}$$

$$\exp\Bigg(\exp\left\{-\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\Bigg)\cdot\exp\left\{-\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot x_{ji}\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot$$

$$\Bigg[\phi^{-1}\Bigg(\exp\left\{-\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\Bigg)\Bigg)\Bigg)\cdot\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}-1}+\Bigg(\phi^{-1}\Bigg(\exp\left\{-\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\Bigg)\Bigg)$$

$$\exp\left\{-\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\Bigg)$$

$$\left(-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot x_{ji}\Bigg)+\Bigg(\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\frac{2\left(\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}\right)\cdot\left(\mathbf{d}_j\right)^2.\left(x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}\right)^4}\cdot x_{ji}\cdot\phi^{-1}\Bigg(\exp\left\{-\left[1+\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\Bigg)\Bigg)$$

$$\left.\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)$$

$$,\left(\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}..x_{ki}\right)^{2}\right)\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}\right.\right.\right..$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}}\right)^{\frac{1}{2}}$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}.x_{ki}\right)\cdot\exp-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot x_{ki}\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)$$

$$\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\Bigg)$$

$$\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot x_{ji}\Bigg)+\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)\cdot\left(\mathbf{d}_k\right)^2.\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^4}\cdot x_{ki}\cdot\phi^{-1}\left(\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\Bigg)^T$$

$$\frac{\partial a_3}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \left(\frac{\partial u_1}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}}, \frac{\partial u_2}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right)$$

$$= \left[\left[\phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) . \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} . \ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \right.$$

$$\left. . \left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{j}} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^2} \right) \right) \right] \left[\phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_k^T \beta_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) . \ \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} . \right.$$

$$\left. \frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} \left[1 + \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} . \left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{-\mathbf{d}_{\mathbf{k}} \left(x_{ki} - \mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^2} \right) \right) \right] \right]$$

$$\frac{\partial B}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} = \frac{1}{2}\left(\frac{\partial a_{11}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot a_{3} + a_{11} \cdot \frac{\partial a_{3}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \right)$$

$$= \frac{1}{2}\left(\left(\frac{\partial a_{1}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot a_{2} + a_{1} \cdot \frac{a_{2}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \right) \cdot a_{3} + a_{1} \cdot a_{2} \cdot \frac{\partial a_{3}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \right), \frac{a_{2}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} = 0$$

$$= \frac{1}{2}\left(\left(\frac{\partial a_{1}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \cdot a_{2} \right) \cdot a_{3} + a_{1} \cdot a_{2} \cdot \frac{\partial a_{3}}{\partial \boldsymbol{\beta}_{\sigma} \partial \boldsymbol{\beta}_{\sigma}} \right)$$

$$= \frac{1}{2}\left(\left(\left(\frac{ -\frac{1}{4}\left(2\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{-\frac{3}{4}} \cdot \left(6\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right)^{2} \right) \cdot \left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{ -\mathbf{d}_{\mathbf{j}} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu} \right) }{ \left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma} \right)^{2} } \right) \right) }{ \left(2\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{3} } \right.\right.\right. \cdot$$

$$\left. \exp\left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) + \left(\frac{1}{ \sqrt{ \left(2\pi \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right)^{3} } } \right)^{\frac{1}{2}} \right)$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}+\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)$$

$$\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left(-\frac{1-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\right)\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\Bigg)$$

$$\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot x_{ji}\Bigg)+\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\frac{2\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)\cdot\left(\mathbf{d}_j\right)^2.\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^4}\right)\cdot\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)$$

$$\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\Bigg)$$

$$,\Bigg(\Bigg(\Bigg(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}..x_{ki}\right)^{2}\right)\cdot\left(-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{k}\left(x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)}{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{3}}.$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{3}}}\right)^{\frac{1}{2}}$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot x_{ki}\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot$$

$$\Bigg[\phi^{-1}\Bigg(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\Bigg)\Bigg)\Bigg)\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}+\Bigg(\phi^{-1}\Bigg(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\Bigg)\Bigg)$$

$$\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\Bigg)$$

$$\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^2}\right)\right)\cdot x_{ji}\Bigg)+\Bigg(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\,\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)\cdot\left(\mathbf{d}_k\right)^2.\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^4}\cdot x_{ki}\cdot\phi^{-1}\Bigg(\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\Bigg)\Bigg)$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\Bigg)^{T}\cdot\left(\rho\left(h\right)^{-1}\right)\cdot$$

$$\cdot\left\{\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.$$

$$\left.\left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{j}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot x_{ji}\right\},$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right.$$

$$\left.\cdot\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot.x_{ki}\right]+\left(\rho\left(h\right)^{-1}\right)\cdot$$

$$\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.$$

$$\left.\left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{j}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot x_{ji}\right),$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right.$$

$$\left.\cdot\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot.x_{ki}\right]$$

$$\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{2}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)}{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{3}}.$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{3}}}\right)^{\frac{1}{2}}\Bigg)$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot x_{ji}\cdot
$$

$$
\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}+\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)\right.\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)
$$

$$
\left.\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\left(-\frac{1-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}\right)\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}-1}\cdot\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\right)
$$

$$
\left.\left(-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot x_{ji}\right)+\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\,\frac{2\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)\cdot\left(\mathbf{d}_j\right)^2.\left(x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^4}\cdot x_{ji}\cdot\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)
$$

$$\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\Bigg)$$

$$,\Bigg(\Bigg(\Bigg(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}..x_{ki}\right)^{2}\right)\cdot\left(-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{k}\left(x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}.$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}}\right)^{\frac{1}{2}}$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)\cdot x_{ki}\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)\cdot$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\right)\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)$$

$$\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)\right)$$

$$\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)\cdot x_{ji}\right)+\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\left(\mathbf{d}_k\right)^2.\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\cdot x_{ki}\cdot\phi^{-1}\left(\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)$$

$$C = \frac{\partial c}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = 0$$

$$\frac{\partial c}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = 0$$

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} = \left(\frac{\partial A}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} B + A \frac{\partial B}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}} \partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right)$$

$$= \left(\frac{\left\{ -\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}} - 1} \right\} \cdot \left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} + \frac{2 \left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right) \cdot \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{4}} \right)}{\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}^{2}} \right.$$

$$\left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} + \frac{2 \left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right) \cdot \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} . \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}} \right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{4}} \right)$$

$$\frac{1}{\left\{\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\right)^{2}\left[\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}-1}\right\}\left[\frac{\left(\mathbf{d}_{j}\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}.\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right]$$

$$\left.\left(\frac{\mathbf{d}_{j}}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}+\frac{2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)\cdot\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}.\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)^{4}}\right)\right)\right)\right)+\left(\frac{2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)\mathbf{d}_{j}}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)^{4}}+\frac{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)\cdot\left(2\mathbf{d}_{j}\cdot\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\right).-2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}.\mathbf{d}_j^{T}\boldsymbol{\beta}\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)^{8}}\right)$$

$$\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}-1}\right\}+\right.$$

$$\left(\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\left(-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}-1\right)\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1-2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\right)}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}-1}\cdot\frac{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}.\left(-\mathbf{d}_{j}\right)^{2}}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\cdot\left(-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{j}\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)\right)\cdot .x_{ji}+$$

$$\left(\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{2\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\left(\mathbf{d}_{\mathbf{j}}\right)^{2}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}}\right)\right).x_{ji}\right)\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\Bigg)$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\right.\right.$$

$$\left.\left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}+\frac{2\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}}\right)\right)+$$

$$\left.\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot.x_{ki}\right)\right]+$$

$$\left(\frac{ \left\{ -\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi \left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma} + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi} - 1} \right\} \cdot \left(\frac{\mathbf{d}_k}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2} + \frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^4} \right) \right) }{ \left\{ \frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma} \left[1 + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}} \right\}^2 } \left(\frac{\mathbf{d}_k}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^2} + \frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^4} \right)$$

$$\frac{1}{ \left\{ \frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma} \left[1 + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}} \right\} } \left\{ \left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\right)^2 \left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma} + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1 - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi} - 1} \right\} \left[\frac{\left(\mathbf{d}_k\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\right)^2} + \frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^4} \right]$$

$$\left(\frac{\left(\mathbf{d}_k\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\right)^2} + \frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^4} \right)\Bigg)\Bigg)\Bigg) + \left(\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)\mathbf{d}_k}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^4} + \frac{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)\cdot\left(2\mathbf{d}_k\cdot\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\right). - 2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi.\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}\left(x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma\right)^8} \right)$$

$$\left(\frac{1}{\left\{ \frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}} \left\{-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}+\right.$$

$$\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}-1\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\right)}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\frac{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\cdot\left(-\mathbf{d}_k\right)^2}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)\cdot.x_{ki}+$$

$$\left(\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\left(\mathbf{d}_k\right)^2\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\right)\right).x_{ki}\right)\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}$$

$$\left[\left(\frac{1}{\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}} \left\{-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}.\right.\right.$$

$$
\Bigg(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} + \frac{2\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}} \Bigg)\Bigg) +
$$

$$
\Bigg(\Bigg\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1} \Bigg\}.\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot .x_{ji} \Bigg)\Bigg]
$$

$$
\frac{1}{2}\Bigg[\Bigg[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}
$$

$$
.\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\Bigg]\Bigg]\Bigg[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\ \exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.
$$

$$
\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}.\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot x_{ki}\Bigg]^{T}
$$

$$\left(\rho\left(h\right)^{-1}\right)\cdot\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.$$

$$\left(\left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{j}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot x_{ji}\right)+$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right.$$

$$\left.\cdot\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot.x_{ki}\right]$$

$$\frac{1}{2}\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right.\right..\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.$$

$$\left.\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot x_{ji}\right]$$

$$,\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right..\ \exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.$$

$$\left.\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot x_{ki}\right]^{T}$$

$$\left(\rho\left(h\right)^{-1}\right)\cdot\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot$$

$$\left(\left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{j}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot x_{ji}\right)+$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right.$$

$$\left.\cdot\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot.x_{ki}\right]+$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\right\}}\left\{-\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left[\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\left(\mathbf{d}_j^T\boldsymbol{\beta}_\sigma\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}-1}\right\}\cdot\right.\right.$$

$$\left.\left(\frac{\mathbf{d}_j}{\left(\mathbf{d}_j^T\boldsymbol{\beta}_\sigma\right)^2}+\frac{2\left(\mathbf{d}_j^T\boldsymbol{\beta}_\sigma\right)\cdot\mathbf{d}_j^T\boldsymbol{\beta}_\xi.\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^T\boldsymbol{\beta}_\sigma\right)^4}\right)\right)+$$

$$\left.\left(\left\{\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}-1}\right\}\cdot\left(-\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^T\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot.x_{ji}\right)\right].$$

$$
\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left[\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\cdot\right.\right.
$$

$$
\left.\left(\frac{\mathbf{d}_{\mathbf{j}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}+\frac{2\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}.\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}}\right)\right)+
$$

$$
\left.\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right\}\cdot\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot.x_{ki}\right)\right].
$$

$$
\cdot\frac{1}{2}\left(\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{2}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{3}}\right.\right.\right.\cdot
$$

$$
\left.\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{3}}}\right)^{\frac{1}{2}}\right)
$$

$$\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\Bigg)$$

$$,\Bigg(\Bigg(\Bigg(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}..x_{ki}\right)^{2}\right)\cdot\left(-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{k}\left(x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}.$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}}\right)^{\frac{1}{2}}$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)\cdot x_{ki}\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)\cdot$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)$$

$$\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)\Bigg)$$

$$\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)\cdot x_{ji}\Bigg)+\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\left(\mathbf{d}_k\right)^2.\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^4}\cdot x_{ki}\cdot\phi^{-1}\left(\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\Bigg)^{T}\cdot\left(\rho\left(h\right)^{-1}\right)\cdot$$

$$
\cdot\left\{\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.
$$

$$
\left.\left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{j}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot x_{ji}\right),
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right.
$$

$$
\left.\cdot\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}.\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot.x_{ki}\right]+\left(\rho\left(h\right)^{-1}\right)\cdot
$$

$$\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.$$

$$\left.\left(\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{j}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot x_{ji}\right),$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right.$$

$$\left.\cdot\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{k}}\left(x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot.x_{ki}\right]$$

$$
\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{2}\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{\mathbf{j}}\left(x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{3}}.
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{3}}}\right)^{\frac{1}{2}}\right)
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}}\right\}\cdot\left(-\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot x_{ji}\cdot
$$

$$
\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}}\right\}+\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}-1}\cdot\left(-\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}}\right\}\right)\right)\right]\cdot\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}-1}+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}}\right\}\right)\right)
$$

$$
\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}}\right\}\cdot\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}\left(-\frac{1-\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}\right)\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1-\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}-1}\cdot\left(-\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\Bigg)
$$

$$
\left(-\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{-\mathbf{d}_j\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^2}\right)\right)\cdot x_{ji}\Bigg)+\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\,\frac{2\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)\cdot\left(\mathbf{d}_j\right)^2.\left(x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu\right)}{\left(\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma\right)^4}\cdot x_{ji}\cdot\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_\xi}}\right\}\right)\right)
$$

$$
\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\Bigg)
$$

$$
,\Bigg(\Bigg(\Bigg(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}..x_{ki}\right)^{2}\right)\cdot\left(-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_{k}\left(x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}.
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}}\right)^{\frac{1}{2}}
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right).
$$

$$
\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right).
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)\right)\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}+\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right.
$$

$$
\left.\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left(-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\right)\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\cdot\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)
$$

$$
\left.\left(-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{-\mathbf{d}_k\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\cdot x_{ji}\right)+\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\,\frac{2\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)\cdot\left(\mathbf{d}_k\right)^{2}.\left(x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}\right)}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{4}}\right)\cdot\phi^{-1}\left(\exp\left\{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)
$$

$$
\left.\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}-1}\right)
$$

Lampiran 15. Turunan Kedua Fungsi *Ln Pairrwise Likelihood* Copula Gaussian terhadap parameter β_ξ

$$\frac{\partial^2 l(\boldsymbol{\beta})}{\partial \beta_\xi \partial \beta_\xi} = \frac{\partial \left(\begin{array}{l} \displaystyle\sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \ln \left(\left\{ \frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}} \left[1 + \mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}} \right\} \exp\left\{ - \left[1 + \mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}} \right\} \right. \\ \left. \left\{ \frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}} \left[1 + \mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\xi}} \right\} \exp\left\{ - \left[1 + \mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\xi}} \right\} \right) \cdot \\ \left[\frac{1}{2} \left[\Phi^{-1} \left(\exp\left\{ - \left[1 + \mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}} \right\} \right) \Phi^{-1} \left[\exp\left\{ - \left[1 + \mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\xi}} \right\} \right] \right]^T \cdot \right. \\ \left. \left(\rho(h)^{-1} \right) \cdot \left[\Phi^{-1} \left(\exp\left\{ - \left[1 + \mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}} \right\} \right) \Phi^{-1} \left[\exp\left\{ - \left[1 + \mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T}\boldsymbol{\beta}_\xi}} \right\} \right] \right] \right] \text{-}0.5 \ln |\rho(h)| \end{array} \right)}{\partial \beta_\xi \partial \beta_\xi}$$

$$Misalkan\quad \frac{\partial l(\boldsymbol{\beta})}{\partial \beta_{\xi}\partial \beta_{\xi}} = \frac{\partial (A.B-C)}{\partial \beta_{\xi}\partial \beta_{\xi}}$$

$$A = \ln\left(\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.$$

$$\left.\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)$$

$$\frac{\partial A}{\partial \beta_{\xi}} = \left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left(\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]\right)\cdot\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)+\right.$$

$$\left.\left(\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left(\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]\right)\cdot\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right]+$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left(\ln\left(\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)\cdot\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)+$$

$$\left(\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left(\ln\left(-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)\right]$$

Misalkan $\frac{\partial A}{\partial\beta_{\xi}\partial\beta_{\xi}}=(x+y)\cdot(w+z)$

maka $\frac{\partial A}{\partial\beta_{\xi}\partial\beta_{\xi}}=\frac{\partial(x+y)}{\partial\beta_{\xi}\partial\beta_{\xi}}+\frac{\partial(w+z)}{\partial\beta_{\xi}\partial\beta_{\xi}}$

$$
\frac{\partial\left(x+y\right)}{\partial\beta_{\xi}\partial\beta_{\xi}}=\left(\frac{\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}^{2}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}
$$

$$
\left(\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]\right)\cdot\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\left(\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]\right)\cdot\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\Bigg)+
$$

$$
\frac{1}{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]}\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\cdot\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}
$$

$$
\frac{\partial(w+z)}{\partial\beta_{\xi}\partial\beta_{\xi}}=\left(\frac{\left\{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}^2}\left\{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\left\{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}
$$

$$
\left(\ln\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]\right)\cdot\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\left(\ln\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]\right)\cdot\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\Bigg)+
$$

$$
\frac{1}{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]}\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\cdot\frac{1}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\xi}}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\xi}}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.
$$

$$
\frac{\partial A}{\partial \beta_{\xi}\partial \beta_{\xi}} = \left(\frac{\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\} \cdot \ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}^{2}} \left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\} + \left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}
$$

$$
\left(\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]\right) \cdot \mathbf{d}_{j}\left(\frac{x_{ji} - \mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\left(\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]\right) \cdot \mathbf{d}_{j}\left(\frac{x_{ji} - \mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\Bigg) +
$$

$$
\frac{1}{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]}\mathbf{d}_{j}\left(\frac{x_{ji} - \mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right) \cdot \frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\xi}}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\xi}}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\xi}}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}
$$

$$
\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left(\ln\left(\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)\right)\cdot\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right)+\right.
$$

$$
\left.\left(\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left(\ln\left(-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot.x_{ki}\right)\right]
$$

$$
\left(\frac{\left\{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}^{2}}\left\{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\left\{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right.
$$

$$
\Bigg(\ln\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]\Bigg)\cdot\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\Bigg(\ln\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]\Bigg)\cdot\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\Bigg)+
$$

$$
\frac{1}{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]}\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\cdot\frac{1}{\left\{\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot
$$

$$
\Bigg[\Bigg(\frac{1}{\left\{\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left[\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]\right)\cdot\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\Bigg)+
$$

$$
\left(\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left[\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right]\right)\cdot\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2}\right)\right)\Bigg]
$$

$$B=\begin{bmatrix}\dfrac{1}{2}\left[\left[\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\dfrac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right],\left[\Phi^{-1}\left[\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\dfrac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right]\right]\right]^{T}. \\ \left(\rho\left(h\right)^{-1}\right)\cdot\left[\left[\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\dfrac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right],\left[\Phi^{-1}\left[\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\dfrac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right]\right]\right]\end{bmatrix}$$

$$\frac{\partial B}{\partial\beta_{\xi}}=\frac{1}{2}\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right].\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.$$

$$\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg]\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right].$$

$$\exp\Biggl\{-\Biggl[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Biggr\}.\Biggl[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\Biggl(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr)\cdot\Biggl(-\mathbf{d}_{k}\Biggl(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr)\Biggr]^{T}.$$

$$\Bigl(\rho\left(h\right)^{-1}\Bigr).\Biggl(\phi^{-1}\Biggl(\exp\Biggl\{-\Biggl[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Biggr\}\Biggr]\Biggr).\exp\Biggl\{-\Biggl[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Biggr\}\Biggl[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.$$

$$\ln\Biggl(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr)\cdot\Biggl(-\mathbf{d}_{\mathbf{j}}\Biggl(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr)\cdot x_{ji}\Biggr]\Biggr)\Biggl[\phi^{-1}\Biggl(\exp\Biggl\{-\Biggl[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Biggr\}\Biggr]\Biggr).\exp\Biggl\{-\Biggl[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Biggr\}.$$

$$\Biggl[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\Biggl(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\Biggl(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr)\cdot\Biggl(-\mathbf{d}_{k}\Biggl(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Biggr)\Biggr)\Biggr)\Biggr]$$

$$\frac{\partial B}{\partial \beta_\xi \partial \beta_\xi} = \frac{1}{2}\left(a_1 \cdot a_2 \cdot a_3^{\ T} + a_1 \cdot^{T} a_2 \cdot a_3\right),\ \text{misalkan } a_1 \cdot a_2 = a_{11} \text{ maka}$$

$$\frac{\partial B}{\partial \beta_\xi \partial \beta_\xi} = \frac{1}{2}\left(\left(\frac{\partial a_{11}}{\partial \beta_\xi \partial \beta_\xi} \cdot a_3 + a_{11} \cdot \frac{\partial a_3}{\partial \beta_\xi \partial \beta_\xi}\right) + \left(\frac{\partial a_{11}}{\partial \beta_\xi \partial \beta_\xi} \cdot a_3 + a_{11} \cdot \frac{\partial a_3}{\partial \beta_\xi \partial \beta_\xi}\right)\right)$$

$$\frac{\partial a_{11}}{\partial \beta_\xi \partial \beta_\xi} = \frac{\partial a_1}{\partial \beta_\xi \partial \beta_\xi} \cdot a_2 + a_1 \cdot \frac{a_2}{\partial \beta_\xi \partial \beta_\xi},\ \frac{a_2}{\partial \beta_\xi \partial \beta_\xi} = 0,\ \text{maka } \frac{\partial a_{11}}{\partial \beta_\xi \partial \beta_\xi} = \frac{\partial a_1}{\partial \beta_\xi \partial \beta_\xi} \cdot a_2$$

$$a_1 = (u_1, u_2)$$

$$\frac{\partial a_1}{\partial \beta_\xi \partial \beta_\xi} = \left(\frac{\partial u_1}{\partial \beta_\xi \partial \beta_\xi}, \frac{\partial u_2}{\partial \beta_\xi \partial \beta_\xi}\right)$$

$$u_1 = a \cdot b \cdot c \cdot d\ \ \text{,misalkan } a \cdot b = k \text{ dan } c.d = l.$$

$$\frac{\partial u_1}{\partial \beta_\xi \partial \beta_\xi} = \frac{\partial k}{\partial \beta_\xi \partial \beta_\xi} l + \frac{\partial l}{\partial \beta_\xi \partial \beta_\xi} k$$

$$= \left(\left(\frac{\partial a}{\partial \beta_\xi \partial \beta_\xi} \cdot b + \frac{\partial b}{\partial \beta_\xi \partial \beta_\xi} \cdot a\right) c \cdot d + \left(\frac{\partial c}{\partial \beta_\xi \partial \beta_\xi} \cdot d + \frac{\partial d}{\partial \beta_\xi \partial \beta_\xi} \cdot c\right) a \cdot b\right)$$

$$a = \left[\phi^{-1} \left(\exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\sigma}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \right) \right.$$

$$\frac{\partial a}{\partial \beta_{\xi}\partial \beta_{\xi}}=\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)^{2}\right)\cdot\ln\left(1+\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)}{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{3}}\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right)^{3}}}\right)^{\frac{1}{2}}\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{j}^{\mathrm{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot$$

$$\mathrm{b} = \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}$$

$$\frac{\partial b}{\partial \beta_{\xi} \partial \beta_{\xi}} = \exp\left\{ -\left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \cdot \ln\left(1 + \mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right) \cdot \left(-\mathbf{d}_{j} \left(\frac{x_{ji} - \mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right)$$

$$c = \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}}$$

$$\frac{\partial c}{\partial \beta_{\xi} \partial \beta_{\xi}} = \ln\left(1 + \mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right) \cdot \left(-\mathbf{d}_{j} \left(\frac{x_{ji} - \mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right)$$

$$d = \ln\left(1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right) \cdot \left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)$$

$$\frac{\partial d}{\partial \beta_{\xi} \partial \beta_{\xi}} = \frac{1}{\left(1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)} \cdot \left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)^{2}$$

$$\frac{\partial u_1}{\partial \beta_\xi \partial \beta_\xi} = \left(\left(\frac{\left(-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)\right)\right)^{-\frac{3}{4}} \cdot \left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)^2\right) \cdot \ln\left(1+\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)^3}\right.\right. \cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)\right)^3}}\right)^{\frac{1}{2}} \cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}\cdot\ln\left(1+\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\mu}}{\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\boldsymbol{\sigma}}\right)\right)$$

$$
\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).
$$

$$
\left.\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ji}+
$$

$$
\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right].\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}
$$

$$
\frac{1}{\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)^{2}
$$

Dengan cara permisalan yang sama pada u_1 maka dapat diperoleh u_2

$$\frac{\partial u_2}{\partial \beta_\xi \partial \beta_\xi} = \left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)^{-\frac{3}{4}}\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)^2\right)\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right)}{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right)^3}\right.\right. \cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}.\right)\right)^3}}\right)^{\frac{1}{2}}\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right).$$

$$
\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg)
$$

$$
\Bigg[\phi^{-1}\Bigg(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\Bigg)\Bigg)\Bigg]\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot
$$

$$
\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg)\Bigg)\Bigg(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg)\Bigg)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg)\cdot x_{ki}+
$$

$$
\Bigg[\Bigg[\phi^{-1}\Bigg(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\Bigg)\Bigg).\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}
$$

$$
\frac{1}{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]}\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)^{2}
$$

$$\frac{\partial a_1}{\partial\beta_\xi\partial\beta_\xi} = \left(\frac{\partial u_1}{\partial\beta_\xi\partial\beta_\xi},\frac{\partial u_2}{\partial\beta_\xi\partial\beta_\xi}\right)$$

$$= \left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)^2\right)\cdot\ln\left(1+\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right)\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)^3}\right.\right.\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)^3}}\right)^{\frac{1}{2}}\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T}\boldsymbol{\beta}_\xi}}\right\}\cdot\ln\left(1+\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right)\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\mu}{\mathbf{d}_j^\mathbf{T}\boldsymbol{\beta}_\sigma}\right)\right)\cdot$$

$$
\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).
$$

$$
\left.\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ji}+
$$

$$
\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right].\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}
$$

$$
\frac{1}{\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)^{2}.
$$

$$\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{2}\right)\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}\cdot\right.\right.$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}}\right)^{\frac{1}{2}}\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}\cdot$$

$$\exp\Bigg\{-\Bigg[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}+\exp\Bigg\{-\Bigg[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\cdot\ln\Bigg(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg)\cdot\Bigg(-\mathbf{d}_k\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg)$$

$$\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\Bigg[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg)\Bigg]\Bigg[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}+\phi^{-1}\Bigg(\exp\Bigg\{-\Bigg[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg)\cdot$$

$$\exp\Bigg\{-\Bigg[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\cdot\ln\Bigg(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg)\cdot\Bigg(-\mathbf{d}_k\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg)\Bigg)\Bigg(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg)\cdot\Bigg(-\mathbf{d}_k\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg)\cdot x_{ki}+$$

$$\Bigg[\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\Bigg[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}\Bigg).\exp\Bigg\{-\Bigg[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\Bigg\}.\Bigg[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}$$

$$\frac{1}{\Bigg[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg]}\cdot\Bigg(-\mathbf{d}_k\Bigg(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\Bigg)\Bigg)^2\cdot x_{ki}\Bigg]$$

$$a_2 = \left(\rho(h)^{-1}\right)$$

$$\frac{\partial a_2}{\partial \beta_\xi \partial \beta_\xi} = 0$$

$$\frac{\partial a_3}{\partial \beta_\xi \partial \beta_\xi} = \Bigg(\phi^{-1} \Bigg(\exp\left\{ -\left[1 + \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi}} \right\} \Bigg) . \exp\left\{ -\left[1 + \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi}} \right\} \left[1 + \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi}} .$$

$$\ln\left(1 + \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\sigma} \right) \right) \cdot \left(-\mathbf{d}_j \left(\frac{x_{ji} - \mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{j}^\mathbf{T} \boldsymbol{\beta}_\sigma} \right) \right) \cdot x_{ji} \Bigg] \Bigg) \Bigg[\phi^{-1} \Bigg(\exp\left\{ -\left[1 + \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi}} \right\} \Bigg) . \exp\left\{ -\left[1 + \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi}} \right\} .$$

$$\left[1 + \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\sigma} \right) \right]^{-\frac{1}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi}} . \ln\left(1 + \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\xi \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\sigma} \right) \right) \cdot \left(-\mathbf{d}_k \left(\frac{x_{ki} - \mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\mu}{\mathbf{d}_\mathbf{k}^\mathbf{T} \boldsymbol{\beta}_\sigma} \right) \right) \Bigg] \Bigg]$$

$$\frac{\partial B}{\partial \beta_{\xi}\partial \beta_{\xi}} = \frac{1}{2}\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{2}\right)\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{3}}\right.\right.\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{3}}}\right)^{\frac{1}{2}}\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)$$

$$
\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot
$$

$$
\left.\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)\left(1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ji}+
$$

$$
\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right].\exp\left\{-\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right.
$$

$$
\frac{1}{\left(1+\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_j^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)^{2}.
$$

$$\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{2}\right)\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}\right.\right.\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}}\right)^{\frac{1}{2}}\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}\cdot$$

$$\left.\exp\left\{-\left[1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right.$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]\left[1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot$$

$$\left.\exp\left\{-\left[1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)\left(1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}+$$

$$\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right].\exp\left\{-\left[1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right.$$

$$\left.\frac{1}{\left[1+\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]}\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)^2\cdot x_{ki}\right]^T\cdot\left(\rho\left(h\right)^{-1}\right)\cdot\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathrm{T}}\boldsymbol{\beta}_{\xi}}}\right\}$$

$$\Bigg[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ji}\Bigg]\Bigg)\Bigg],\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\Bigg[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg).$$

$$\exp\Bigg\{-\Bigg[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}.\Bigg[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{k}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}\Bigg]+$$

$$\Bigg[\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\Bigg[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg).\exp\Bigg\{-\Bigg[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}.\Bigg[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.$$

$$\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ji}\Bigg],\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\Bigg[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T}\beta_{\sigma}}\right)\Bigg]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg)$$

$$\exp\Bigg\{-\Bigg[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}.\Bigg[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\Bigg]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{k}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}\Bigg]^{T}\cdot\left(\rho\left(h\right)^{-1}\right)\cdot$$

$$
\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{2}\right)\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{3}}\cdot\right.\right.
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{3}}}\right)^{\frac{1}{2}}\cdot
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ji}\cdot
$$

$$\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right)$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}\right\}\right).$$

$$\left.\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right)\right)\left(1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot x_{ji}+$$

$$\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right.\right..\exp\left\{-\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}}}$$

$$\frac{1}{\left(1+\mathbf{d}_j^{T}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right)}\cdot\left(-\mathbf{d}_j\left(\frac{x_{ji}-\mathbf{d}_j^{T}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_j^{T}\boldsymbol{\beta}_{\sigma}}\right)\right)^2\cdot x_{ji}\ ,$$

$$
\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{2}\right)\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}\right.\right.\cdot
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}}\right)^{\frac{1}{2}}\cdot
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}\cdot
$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right)$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\right]\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right)\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right)\Bigg)\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right)\cdot x_{ki}+$$

$$\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right\}.\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi}}\right.\right.$$

$$\left.\frac{1}{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right]}\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\mu}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_\sigma}\right)\right)^2\cdot x_{ki}\right]$$

$$\frac{\partial C}{\partial\beta_\xi\partial\beta_\xi}=0$$

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \beta_{\xi}\partial \beta_{\xi}} = \frac{\partial (A \cdot B - C)}{\partial \beta_{\xi}\partial \beta_{\xi}}, \; \frac{\partial C}{\partial \beta_{\xi}\partial \beta_{\xi}} = 0 \text{ maka}$$

$$= \left(\frac{\partial A}{\partial \beta_{\xi}\partial \beta_{\xi}} B + A \frac{\partial B}{\partial \beta_{\xi}\partial \beta_{\xi}} \right)$$

$$= \left(\frac{\left\{ \left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \cdot \ln \left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} \right) \right]}{\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\}^{2}} \left\{ \left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} + \left\{ \left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \right.$$

$$\left(\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]\right)\cdot\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\left(\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]\right)\cdot\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\Bigg)+$$

$$\frac{1}{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]}\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\cdot\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left(\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)\cdot\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)+$$

$$\left(\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left(-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)\Bigg]$$

$$
\left(\frac{\left\{ \left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \cdot \ln \left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \right]}{\left\{ \frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\}^2} \left\{ \left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} + \left\{ \left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \right.
$$

$$
\left(\ln \left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \right] \right) \cdot \mathbf{d}_k \left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \left(\ln \left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \right] \right) \cdot \mathbf{d}_k \left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \Bigg) +
$$

$$
\frac{1}{\left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \right]} \mathbf{d}_k \left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \cdot \frac{1}{\left\{ \frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\}} \left\{ \left[\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki} - \mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^2} \right) \right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}} \right\} \cdot
$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]\right)\cdot\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)+\right.$$

$$\left.\left(\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]\right)\cdot\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\cdot.x_{ji}\right)\right]\cdot$$

$$\frac{1}{2}\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.\right.$$

$$\left.\left.\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right]\right]\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right..$$

$$\left.\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{k}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}\right]^{T}\cdot$$

$$\left(\rho\left(h\right)^{-1}\right).\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.$$

$$\left.\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ji}\right]\Bigg),\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\right.$$

$$\left.\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{k}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right]+$$

$$
\left[\left(\frac{1}{\left\{ \frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\}} \left\{ \left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \cdot \left(\ln \left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right] \right) \cdot \mathbf{d}_{j} \left(\frac{x_{ji} - \mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right) + \right.
$$

$$
\left. \left(\left\{ - \left[1 + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} \right) \right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi}}} \right\} \cdot \left(\ln \left[\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}}} + \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\xi} \left(\frac{x_{ji} - \mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{j}}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right] \right) \cdot \mathbf{d}_{j} \left(\frac{x_{ji} - \mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{j}^{\mathbf{T}} \boldsymbol{\beta}_{\boldsymbol{\sigma}} \right)^{2}} \right) \right) \right]
$$

$$
\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}}\left\{\left[\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left(\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{k}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)\right)\cdot\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\left(\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}\right)^{2}}\right)\right)+
$$

$$
\left(\left\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\left(\ln\left(-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{k}}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)\right]
$$

$$\frac{1}{2}\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{2}\right)\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{3}}\right.\right.\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{3}}}\right)^{\frac{1}{2}}\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).$$

$$\left.\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ji}+$$

$$\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right].\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}$$

$$\frac{1}{\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)^{2}\cdot x_{ji}\ ,$$

$$
\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{2}\right)\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}\cdot\right.\right.
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{3}}}\right)^{\frac{1}{2}}\cdot
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}\cdot
$$

$$
\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]\left[\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot\right.
$$

$$
\left.\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\right)\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}+
$$

$$
\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right].\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}
$$

$$
\left.\frac{1}{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]}\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)^{2}\cdot x_{ki}\right]^{T}\cdot\left(\rho\left(h\right)^{-1}\right)\cdot\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}
$$

$$\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot x_{ji}\Bigg]\Bigg),\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg).$$

$$\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}.\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_{k}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot x_{ki}\Bigg]+$$

$$\Bigg[\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg).\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}.\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.$$

$$\ln\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot x_{ji}\Bigg],\Bigg[\phi^{-1}\Bigg(\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{d_{k}^{T}\beta_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}\Bigg)$$

$$\exp\Bigg\{-\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\Bigg\}.\left[1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}.\ln\left(1+\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot\left(-\mathbf{d}_{k}\left(\frac{x_{ki}-\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\mu}}{\mathbf{d}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\beta}_{\sigma}}\right)\right)\cdot x_{ki}\Bigg]^{T}\cdot\left(\rho\left(h\right)^{-1}\right)\cdot$$

$$\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{2}\right)\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{3}}\right.\right.\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)+\left(\frac{1}{\sqrt{2\pi\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)^{3}}}\right)^{\frac{1}{2}}\cdot$$

$$\exp\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ji}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot$$

$$
\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)
$$

$$
\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).
$$

$$
\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{j}\left(\frac{x_{ji}-\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{j}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg)\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)+
$$

$$
\Bigg[\Bigg[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}
$$

$$
\frac{1}{\left(1+\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}\cdot\left(-\mathbf{d}_{\mathbf{j}}\left(\frac{x_{ji}-\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)^{2}.
$$

$$
\left(\left(\frac{-\frac{1}{4}\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{-\frac{3}{4}}\cdot\left(6\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)^{2}\right)\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)}{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{3}}\cdot\right.\right.
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)+\left(\frac{1}{\sqrt{\left(2\pi\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\right)^{3}}}\right)^{\frac{1}{2}}\cdot
$$

$$
\exp\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.x_{ki}\right)\cdot\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right).
$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}+\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right]\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}+\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\cdot$$

$$\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\cdot\ln\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\Bigg)\left(1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)\cdot x_{ki}+$$

$$\Bigg[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}\right)\right].\exp\left\{-\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}\right\}.\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]^{-\frac{1}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}}}$$

$$\frac{1}{\left[1+\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\xi}\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right]}\cdot\left(-\mathbf{d}_k\left(\frac{x_{ki}-\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}}{\mathbf{d}_k^{\mathbf{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}}\right)\right)^{2}\Bigg]$$

Lampiran 16. Jarak Euclidean 11 Pos Hujan

	Gemarang	Guyung	Karangjati	Kedungbendo	Kedunggalar	Kendal	Kricak	Mantingan	Mardisari	Papungan	Paron
Gemarang	0	0,118	0,255	0,176	0,055	0,182	0,021	0,217	0,051	0,014	0,050
Guyung		0	0,208	0,178	0,138	0,134	0,129	0,287	0,077	0,129	0,070
Karangjati			0	0,103	0,304	0,340	0,276	0,470	0,209	0,256	0,219
Kedungbendo				0	0,231	0,308	0,198	0,393	0,143	0,173	0,156
Kedunggalar					0	0,154	0,035	0,165	0,095	0,062	0,088
Kendal						0	0,175	0,223	0,176	0,195	0,163
Kricak							0	0,195	0,070	0,027	0,067
Mantingan								0	0,260	0,220	0,251
Mardisari									0	0,058	0,013
Papungan										0	0,060
Paron											0

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Nomor : UM.001/372/KPD/V/2016 Jakarta, 18 Mei 2016
Sifat : Penting
Perihal : Data Curah Hujan Harian Kab. Ngawi

Yth. Ketua Jurusan Statistika
Fakultas Matematika dan Ilmu Pengetahuan Alam
Institut Teknologi Sepuluh Nopember - Surabaya

Up. Dr. Sutikno, M.Si

di
Tempat

Bersama ini kami sampaikan data pos hujan kerjasama untuk Kabupaten Ngawi Propinsi Jawa Timur sesuai dengan permohonan Saudara dengan catatan sebagai berikut:

1. Data curah hujan tersedia mulai periode Januari 1980 sampai dengan Desember 2015 untuk seluruh lokasi kecuali pos hujan kerjasama Ngale;
2. Data curah hujan harian yang tersedia dalam satuan milimeter (mm);
3. Data hanya digunakan untuk kepentingan penelitian dan penyusunan tugas akhir dengan topik penelitian "Spatial Extreme Value untuk Prediksi Curah Hujan Ekstrem, dengan studi kasus di wilayah Kabupaten Ngawi". Kami minta diberikan fotocopy hasil penelitiannya;
4. Data tidak bisa digunakan oleh pihak lain.

Demikian disampaikan. Atas kerjasamanya diucapkan terima kasih.

KEPALA PUSAT DATABASE

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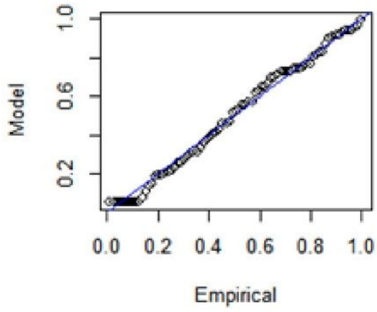
NIP. 19590425 198503 1 001

Lampiran 18. *Probability plot* 10 Pos Hujan



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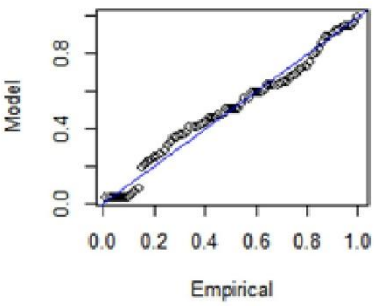

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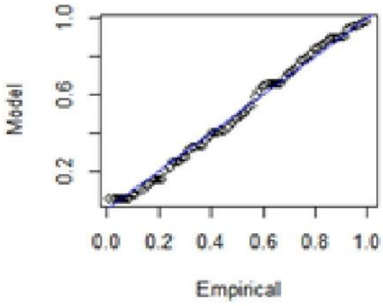

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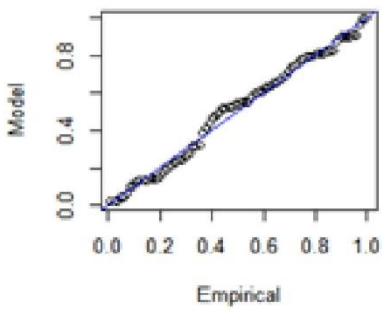

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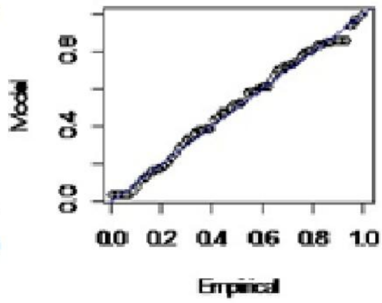

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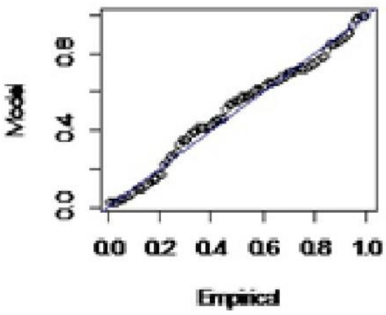

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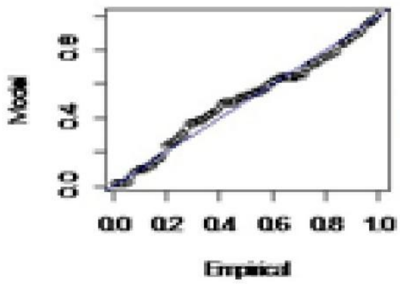
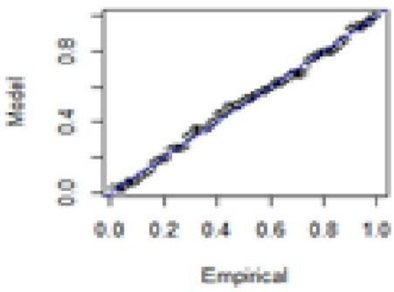
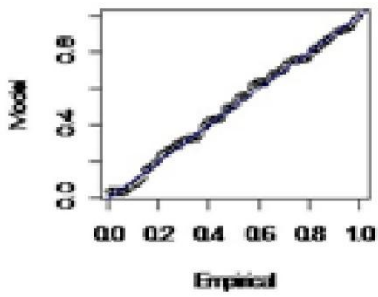


Mantingan



Mardisari



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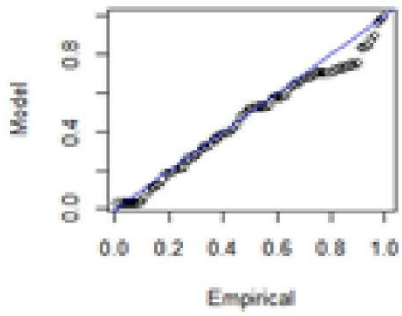

Papungan